REVIEWS

Fair division, STEVEN BRAMS and ALAN TAYLOR. Cambridge University Press, 1996, 272 + xiv pages.

A very common situation confronted in the real world is that of sharing a 'pie' among a few people, each of whom benefits from getting as big a piece as possible. In real life, pies can be divided in many ways; for example, by means of force: one of the parties just grabs the whole pie. But there are also more civilized ways to divide pies; actually, many of our social, economic, legal and political institutions, are kept busy finding ways in which to share pies. In those institutions, interests and various forms of power play key roles. For example, in bargaining, the ability of the parties to credibly threaten the opponent, the degree of their impatience, level of sophistication, etc., are key factors in determining the final outcome.

There is another important element, however, that affects division of pies – 'fairness'. It does so in a variety of ways. First, fairness is a major consideration of an outsider (an arbitrator) assigned to divide a pie. Second, since considerations of fairness affect the preferences of the parties involved it also affects their behavior when applying the mechanisms which are supposed to determine how the pie is divided. For example, a bargainer who is offered what he considers an unfair deal may be willing to sacrifice his own resources in order to punish the offerer. And third, fairness is often considered a legitimate argument when bargaining.

Much of the literature on 'fairness' revolves around the question, 'What is a fair division?' The book under consideration is not about this question. The relevant situations to which the book refers are those in which claims for fairness of the type 'I should get more since I am poor' or, 'I should get more since I have contributed more to obtaining the pie' are irrelevant. In such symmetric situations, the notion of a fair division of a pie is relatively simple: we have strong intuitions that the pie should be divided equally.

So, what is there left to discuss? The problem is that the parties involved may nonetheless retain different views about what constitutes an 'equal' share of the pie. The pie is usually not a chocolate cake but some collection of items evaluated differently by different people. Their diverse views may be an outcome of objectively different values held by the parties, or subjectively disparate perceptions of the components of the pie. In any case, the determination of what makes up an equal division of a pie may be far from trivial.

Concerning situations like this, where the pie is not homogeneous, the book discusses two axioms regarding the notion of 'fair division':

1) Non-envy: no agent prefers a slice that someone else receives.
2) Proportionality: in n-agent situations, each agent gets what he considers at least 1/n of the pie.

The book closely examines a family of mechanisms in the spirit of the 'divide-and-choose' mechanism that implements divisions satisfying at least one of these principles. The history of the divide-and-choose mechanism is described in the book. Let me review this famous mechanism, applicable in the two-agent case. The rules of the game are: one of the agents (the divider) has to divide the pie into two slices, and the other (the chooser) has to choose one of the two slices left by the divider.

Assuming that the divider thinks that the chooser is rational, and assuming the divider knows the way the chooser evaluates the various parts of the 'pie', it is impossible for the chooser not to evaluate the two cut slices as being equal and it is impossible for the divider to get less than what he considers to be one half of the pie. To see this, assume that the divider cuts the pie into the two pieces, A and B, and that the chooser strictly prefers A. The divider will do better by selecting another division in which a small part of A is transferred to B, creating the division A' and B'. The chooser still selects A' and the divider gets a slightly larger slice B'. Thus, it must be that the chooser is indifferent between A and B. If the divider values B as strictly worse than A, he will do better to remove a little from A, so that the chooser will choose B'; leaving the divider with a piece which is still better than B.

Note that in the above arguments, we have used several assumptions: first, we assumed that the union of two non-empty slices is strictly better than each of the slices taken separately. Second, we assumed some sort of continuity – the existence of small slices to be transferable from one piece to another in order to carry out the 'profitable deviation'. And most importantly, the entire argument is based on the assumption that the two agents behave rationally.

The mechanism is beautiful. It has the flavor of a puzzle, which may

be the reason why it attracts so much attention from mathematicians. But it is only the starting point of the book, since the equivalence of the two principles of non-envy and proportionality does not hold for more than two players, and it is not trivial to extend the mechanics 'to work' for more than two agents. Much of the book is devoted to the discussion of several mechanisms to satisfy one of the two principles in a situation involving three or more agents.

The book, therefore, deals with procedures for obtaining fair divisions. The approach is game theoretical. Thus, the book can be thought of as the analysis of a particular (and important!) special case of the implementation problem, one that seeks out mechanisms for implementing desirable social targets.

Steve Brams has a great record as a writer. Among his many books, one can identify at least three themes. The first group, including, most noticeably, *Biblical Games*, has served an important role in popularizing game theory. Others such as *Game Theory and Politics* and *Game Theory and the National Security* (co-authored with Marc Kilgour), bridge the gap between the growing literature on game theory and political science. The third group of books, and particularly *Approval Voting* (coauthored with Peter Fishburn), advocate the use of certain mechanisms (a voting system in this case) that are based on game theoretical ideas.

This new book falls within the third category. It is not a 'light' book. Reading it requires much attention. The arguments are quite complicated. But, the book has clear pretensions toward being directly useful. There is no question that game theory in general, and the implementation literature in particular comprises a rich body of ideas that seem to inspire mechanism designers. But the authors are not just interested in providing general ideas and analytical frameworks. They have in mind the clear aim of offering mechanisms to be adopted in real life by lawyers, mediators and ordinary people. This, I have to say, is a target that makes me uncomfortable.

At the outset, let me stress the point that, indeed, the 'general public' perceives such suggestions in a quite simple-minded way. Steve Brams kindly provided me with a piece from the popular press written by K. C. Cole, a science reporter for the Washington edition of the Los Angeles Times, dated 26 April, 1996. Cole perceives mathematicians as problem-solvers with considerable power to solve very complicated problems. She says: 'Mathematics may now be used to solve a problem that has tormented people since the dawn of humanity: how to divide things fairly'. Or, in another quote: 'Mathematics is invading political science in an attempt to find rational approaches to complex, often highly emotional questions'. The most striking description of the significance of the research is given in the article as a quotation from John Ledyard (a social sciences professor at Caltech): 'Philosophers have argued about fairness

for thousands of years. What's different now is that we have a formal mathematical structure. That takes it out of the ideology debate. There's science here'.

These quotations are supported in the article by the idea that mathematics can solve problems in which the emotions and ideology played a part. I find this perception of game theory rather alarming. Mathematicians, social scientists, economists and political scientists cannot have solutions to emotional problems, nor provide objective 'scientific' substitutes for ideology. They can supply arguments. They can clean up inconsistent intuitions. That is all.

I am very much aware of the risk that game-theoretical ideas, like those in this book, will supply their users with arguments that seemingly bear the hallmark of scientific truth and are free of ideology. Designers of systems always look for authority to justify their proposals and they may find it convenient to refer to this book even if they do not understand it, as a source of support for ideas motivated by very different reasons. In my opinion, the book, like any other formal game-theoretical work, serves no more (but certainly no less!) purpose than to illuminate some ideas and, at best, as conceptual inspiration for the construction of new mechanisms.

Let me briefly discuss three other points.

(a) One important issue, ignored in the book, relates to the complexity of the procedures. If a procedure is complicated, the result of its application may be critically affected by the sophistication of the agents. A person may thus feel that he was treated unfairly with a division resulting from a mechanism that was meant to be fair. He may feel that the procedure was too intricate and that he was not sophisticated enough to behave optimally. Issues related to the complexity of mechanisms have hardly been dealt with in the implementation literature, and more theoretical work is required to build the tools necessary for evaluating the complexity factors. In the meantime, whoever is thinking of using such mechanisms to divide a pie, land, or inheritance, should be aware of this major problem.

(b) The book makes much use of game-theoretic tools. One of the main achievements of game theory is the clear distinction between the rules of the situation, the preferences of the players, and the strategies. The authors describe this distinction in Section 1.5. But then, at the bottom of page 23 they state: 'For more complicated schemes, however, it is easier to understand a procedure if one is given the strategies at the same time as one is given the rules'. I strongly disagree. One cannot talk about strategies before defining the game form. Furthermore, the rules of the mechanism consist of the instructions given to the agents; if they are too complicated to be described in this book, how can one expect ordinary contenders in a divorce proceeding, for example, to understand them?

(c) The linkage between the first seven chapters of the book, dealing with the 'partition of the pie', and the other three chapters dealing with models of bargaining, auctions and elections is not sufficiently clear. The last three chapters disturb the homogeneous content of the main body of the book, and will most likely be ignored by its readers.

To summarize, though I disagree with the spirit of the book and its practical intentions, I find it to be very valuable. It will surely contribute towards spreading game-theoretical ideas to wider circles, and it will serve as a good reference work for scholars interested in these types of problems and models.

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