Equilibrium in a Civilized Jungle

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ABSTRACT: The *jungle model* with an equal number of agents and objects is enriched by adding a *language*, which is a set of orderings over the set of agents. An assignment of an agent to an object is *justified* within a group of agents if there is an ordering according to which that agent is the best-suited in the group. A *civilized equilibrium* is an assignment such that every agent is the *strongest* in the group of agents consisting of himself and those who wish to be assigned to the object and can be justified within this group.

We present (i) conditions under which the equilibrium in a civilized jungle is identical to the jungle equilibrium; (ii) a connection between the power relation and the language that is essentially necessary and sufficient for the existence of a Pareto efficient civilized equilibrium; and (iii) a counterpart to the second welfare theorem.

KEYWORDS: Jungle Equilibrium, justifiability, civilized equilibrium, welfare theorems.
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1. Introduction

Consider a society consisting of an equal number of agents and objects. An agent has preferences over the objects and there are no externalities. Each agent is to be exclusively assigned to a single object. The agents are ranked by a *power relation*. Power is not necessarily physical strength but can be, for example, social status or seniority. Up to this point, we have described the *jungle model* à la Piccione and Rubinstein (2007) adapted to the object assignment model of Shapley and Scarf (1974).

In a *civilized jungle*, the exercise of power requires some socially legitimate justification. We enrich the jungle model with a *language* that specifies the legitimate criteria that can be used to justify the assignment of an agent to an object. For example, a legitimate criterion might rank the agents according to wealth, intelligence, or education level. The assignment of an agent to an object is justifiable if the agent is uniquely best-suited–according to one of the orderings in the language–from among the set consisting of himself and those who prefer the object to their own assigned object. Thus, when an agent claims that he should have been assigned to a particular object, he must justify his claim using a criterion according to which not only is he better-suited than the agent who is assigned to the object but also that he is better-suited than any other agent who wishes to be assigned to the object.

As is often the case in real life, an agent may self-servingly adopt a criterion that justifies assigning himself to his favorite object. He might justify his claim based on his wealth on one occasion and based on his intelligence on another. An agent wishing to be assigned to an object can use any criterion, an assumption that makes sense in situations where the objects differ but are nonetheless in the same category (such as office space, equally ranked positions in an organization, and time slots for lectures). This as-

sumption would not make sense, for example, in modeling the assignment of hierarchical positions in an organization. Leadership skills might be a criterion for a high-ranking position, but are irrelevant in the case of a low-ranking position, while obedience may be a reasonable criterion for a low-ranking position, but is irrelevant in the case of a high-ranking position.

The proposed solution concept is *civilized equilibrium* (C-equilibrium), which is an assignment such that each agent:

(i) is justifiable within the group consisting of himself and the agents who envy him,

(ii) is stronger than any other agent who is justifiable within the same group.

The C-equilibrium is related to the Jungle Equilibrium, which is an assignment such that if an agent envies another then the latter agent is stronger than the former agent according to the power relation. In a civilized jungle, the language restricts the use of power by determining what can be viewed as a justifiable claim for or against an assignment. When an agent wishes to be assigned to an object that another agent is assigned to, it is not enough that he be stronger; his claim must be justifiable as well. In equilibrium, an agent who is assigned to an object must be the strongest agent in the group of agents consisting of himself and those who envy him and who can justify their claim within this group. Since justifiability is a prerequisite for the use of power, we refer to our equilibrium notion as "civilized equilibrium".

Before getting into the model some words about the motivation of the paper. It is our view that the standard models in economic theory (whether they deal with markets, games or choice problems) lack a critical real-life component, namely the language used by economic agents to communicate, to formulate their rules of behavior, or to maintain

social norms.[1] Our first motivation is to incorporate this approach into the analysis of a jungle model by enriching it with a language that is used by the agents to justify their claims for being assigned to an object.

The second motivation is related to the rhetoric of economic theory. The jungle model was intended to be–at least in part–a critique of the rhetoric used in standard economic theory which is used to extol the market system by way of the welfare theorems. Piccione and Rubinstein (2007) argued that one can in order to employ similar rhetoric to extol the jungle system.

Our results show that civilizing a jungle does not necessarily preserve the existence and efficiency of equilibrium. We observe that the first welfare theorem often fails in a civilized jungle. Therefore, one main focus of the analysis is to find when the equilibrium in a jungle is preserved as a C-equilibrium in a civilized jungle. The results indicate that in a civilized jungle, in order to achieve harmony in the form of a Pareto efficient equilibrium, the power relation should respect the language in a specific way; otherwise, chaos or inefficiency might prevail.

2. The civilized jungle and the civilized equilibrium

A **civilized jungle** is a tuple $\langle N, X, (\succsim^i)_{i\in N}, \trianglerighteq, \mathscr{L}\rangle$. The set of *agents* is $N = \{1,\ldots,n\}$ and the set X consists of n *objects*. Each agent i has a strict preference relation $\succsim^i$, which is a complete, transitive, and antisymmetric binary relation over X. The **power relation** $\trianglerighteq$ is a strict ordering over N. The statement $i \triangleright j$ means that agent i is **stronger than** agent j. The **language** $\mathscr{L}$ is a set of complete and transitive (but not necessarily antisymmetric)

[1] See Rubinstein (1978) and Rubinstein (2000, Chapter 4) who suggests the requirement that an agent's preferences should be *definable* in a given language.

binary relations over the set of agents N. Let $\mathscr{L} = \{\succeq_\lambda\}_{\lambda\in\Lambda}$ where Λ is the index set of $\mathscr{L}$'s members. The set $\mathscr{L}$ consists of the criteria that can be used to justify the choice of an agent from within a **group** that is a nonempty subset of agents. We refer to a civilized jungle without a language as a **jungle**.

An **assignment** $(x^i)_{i\in N}$ maps each agent exclusively to an object. For brevity, we write $\mathbf{x}$ instead of $(x^i)_{i\in N}$. A **jungle equilibrium** is an assignment satisfying that there are no agents i and j such that $x^j \succ^i x^i$ and $i \rhd j$. Unlike in the model of the jungle, the use of the power relation in the civilized jungle is restricted such that a stronger agent can exercise his power in order to be assigned to an object only if he can justify being assigned to it by one of the criteria recognized as legitimate in the civilized jungle. An agent i is **justifiable by** $\succeq_\lambda$ from within the group I if he is the *unique* maximizer of $\succeq_\lambda$ from within I. An agent i is **justifiable** in group I if he is **justifiable by** $\succeq_\lambda$ from within the group I for some $\succeq_\lambda \in \mathscr{L}$. Let $J_{\mathscr{L}}(I)$ be the set of agents justifiable in I. By definition, $J_{\mathscr{L}}(\{i\}) = \{i\}$.

A candidate for the solution concept of a *civilized equilibrium* is an assignment. For an assignment $\mathbf{x}$, an agent j **envies** agent i if $x^i \succ^j x^j$. We denote the group consisting of agent i and all the agents who envy him by $E(\mathbf{x}, i)$.

Definition 1 An assignment $\mathbf{x}$ is a **civilized equilibrium** (C-equilibrium) if each agent i is the $\trianglerighteq$-strongest agent in $J_{\mathscr{L}}(E(\mathbf{x}, i))$.

2.1 Dichotomous languages

A dichotomous language consists of properties (unary relations) that an agent may or may not have. Formally, a **dichotomous language** consists of orderings of N with two (nonempty) indifference sets, a top one and a bottom one, where every agent in the top set is superior to every agent in the bottom set. For each $\lambda \in \Lambda$, we identify the ordering

$\geq_\lambda$ by means of a property λ, in the sense that agent i has property λ if he is in the top set of $\geq_\lambda$ and fails to have property λ if he is in the bottom set of $\geq_\lambda$. A dichotomous language can be represented as a profile $(\phi^i)_{i\in N}$ where ϕ^i is the set of properties in Λ that agent i has. Then, the statement "agent i is justified by λ in group I" means that i is the unique agent in I for whom $\lambda \in \phi^i$. That is, the property λ makes him "special" within the group I.

2.2 Examples

Example A (Restrictive languages) Consider a civilized jungle with a restrictive language $\mathscr{L}$ consisting of a single strict ordering $\geq$ over N. Then, the unique C-equilibrium is obtained independently of the power relation, by running the serial dictatorship according to $\geq$, and therefore it is Pareto efficient.

Example B (The power relation is a member of the language) Consider a civilized jungle in which the power relation $\trianglerighteq$ is a member of $\mathscr{L}$. Then, the assignment $\mathbf{x}$ obtained by running the serial dictatorship according to $\trianglerighteq$ is a Pareto efficient C-equilibrium. To see this, note that if an agent i envies j, then $j \rhd i$. Therefore, since $\trianglerighteq$ is a member of $\mathscr{L}$, j is justified in $E(\mathbf{x}, j)$ by $\trianglerighteq$. It follows that j is the $\trianglerighteq$-strongest agent in $J_{\mathscr{L}}(E(\mathbf{x}, j))$. Indeed, $\mathbf{x}$ is the unique C-equilibrium. Let $\mathbf{y}$ be another assignment and let i be the strongest agent who envies a weaker agent j. Then, i is justified in $E(\mathbf{y}, j)$ by $\trianglerighteq$ since i is the $\trianglerighteq$-strongest agent in $E(\mathbf{y}, j)$ and $\trianglerighteq$ is a member of $\mathscr{L}$. Therefore, $\mathbf{y}$ is not a C-equilibrium.

Example C (Identical preferences) Assume that all agents share the same preferences $a_1 \succ a_2 \succ \cdots \succ a_n$. Suppose that the language $\mathscr{L}$ contains at least one strict ordering. This guarantees that for any group I the set $J_{\mathscr{L}}(I)$ is not empty. Then, inductively choose

the sequence of agents such that i_l is the $\trianglerighteq$-strongest agent in $J_{\mathscr{L}}(N \setminus \{i_1, \ldots, i_{l-1}\})$. The assignment of i_l to a_l is the unique C-equilibrium.

Example D (Justification by "I am who I am") Consider a civilized jungle with the dichotomous language $\phi^i = \{m^i\}$ for every $i \in N$. The statement m^i stands for "my name is i". Such a civilized jungle is extremely permissive in the sense that every agent i can justify being assigned to any object by arguing that he is the unique agent who deserves to be assigned to it by the criterion m^i. Since every agent is justifiable in every group of agents, the unique C-equilibrium is obtained by running serial dictatorship according to the power relation.

Example E (Nested dichotomous languages) Consider a dichotomous language where agents' sets of properties are *nested*, in the sense that there is an ordering $i_1, \ldots, i_n$ of the agents such that $\phi^{i_n} \subset \cdots \subset \phi^{i_2} \subset \phi^{i_1}$. For each preference profile and independently of the power relation, the associated civilized jungle has a unique C-equilibrium obtained by running serial dictatorship according to the ordering $i_1, \ldots, i_n$.

3. The C-equilibrium and the Jungle Equilibrium

As discussed in the introduction, C-equilibrium is related to the Jungle Equilibrium. The key difference between them is that in a C-equilibrium if an agent makes a claim on a different object, then not only must he be stronger than the current agent assigned to the object, he must also provide a justification. However, we will see below that if the power relation respects the language in a specific way, then the Jungle Equilibrium remains a C-equilibrium and under additional conditions is even the unique C-equilibrium.

Two properties of the power relation–given the language–turn out to be critical.

Definition 2 A power relation $\trianglerighteq$ is **weakly $\mathscr{L}$-concave** if for every $i, j \in N$, we have $i \triangleright j$ whenever for every $\succeq_\lambda \in \mathscr{L}$ there exists $i_\lambda \in N \setminus \{j\}$ such that $i_\lambda \succeq_\lambda j$ and $i \triangleright i_\lambda$. A power relation $\trianglerighteq$ is **strongly $\mathscr{L}$-concave** if for every $i, j \in N$, we have $i \triangleright j$ whenever for every $\succeq_\lambda \in \mathscr{L}$ there exists $i_\lambda \in N \setminus \{j\}$ such that $i_\lambda \succeq_\lambda j$ and $i \trianglerighteq i_\lambda$.

A power relation is $\mathscr{L}$-concave if it respects the language in the following sense: Agent j cannot be stronger than agent i if for each criterion, agent i can point to an agent who is weaker than himself and at least as suited as j according to the criterion. As explained by Richter and Rubinstein (2019), $\mathscr{L}$-concavity is not simply a technical condition and is closely related to standard notions of convexity and concavity. In our setting, $\mathscr{L}$-concavity represents an intuitive (though not necessarily realistic) relationship between the power relation and the language.

The weak $\mathscr{L}$-concavity of the power relation–a weaker version of the strong $\mathscr{L}$-concavity defined by Richter and Rubinstein (2019)–is the restriction under which the Jungle Equilibrium is a C-equilibrium. Recall that this assignment always exists and is Pareto efficient. The strong $\mathscr{L}$-concavity of the power relation guarantees that the Jungle Equilibrium will be the unique C-equilibrium in a civilized jungle with a language of strict orderings.

Proposition 1 *Let $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$ be a civilized jungle with a weakly $\mathscr{L}$-concave power relation.*

(i) The Jungle Equilibrium of $\langle N, X, (\succsim^i), \trianglerighteq\rangle$ is a C-equilibrium.

(ii) If $\mathscr{L}$ is a language of strict orderings and $\trianglerighteq$ is a strongly $\mathscr{L}$-concave power relation, then the C-equilibrium is unique.

Proof. To prove (i), let $\mathbf{x}$ be the assignment obtained by running the serial dictatorship according to $\trianglerighteq$. Then, for each $j \in N$, $E(\mathbf{x}, j) \subseteq \{i | j \trianglerighteq i\}$. To show that $\mathbf{x}$ is a C-equilibrium, assume by contradiction that there is an agent j such that $j \notin J_{\mathscr{L}}(E(\mathbf{x}, j))$. Then, for each $\lambda \in \Lambda$, there exists $j_\lambda \in E(\mathbf{x}, j) \setminus \{j\}$ such that $j_\lambda \geq_\lambda j$ and $j \triangleright j_\lambda$. Therefore, by the weak $\mathscr{L}$-concavity of $\trianglerighteq$, we get $j \triangleright j$.

To prove (ii), suppose that $\mathbf{y}$ is another C-equilibrium. Then, there exists $i, j \in N$ such that $i \triangleright j$ and i envies j in $\mathbf{y}$. Since $\mathbf{y}$ is a C-equilibrium, $i \notin J_{\mathscr{L}}(E(\mathbf{y}, j))$. Therefore, for each $\lambda \in \Lambda$, there exists $j_\lambda \in J_{\mathscr{L}}(E(\mathbf{y}, j))$ such that $j_\lambda \geq_\lambda i$. Since $\mathbf{y}$ is a C-equilibrium, then $j \trianglerighteq j_\lambda$ for every $\lambda \in \Lambda$. However, in that case, the strong $\mathscr{L}$-concavity of $\trianglerighteq$ implies that $j \triangleright i$, a contradiction. □

A weaker version of weak $\mathscr{L}$-concavity is $\mathscr{L}$*-reflectivity* which only requires that if an agent i is better-suited than agent j according to all the criteria in $\mathscr{L}$, then i must be stronger than j. Formally, $\trianglerighteq$ is **$\mathscr{L}$-reflective** if for every $i, j \in N$, whenever $i >_\lambda j$ for every $\geq_\lambda \in \mathscr{L}$, we have $i \triangleright j$. The following example demonstrates that if the power relation is not weakly $\mathscr{L}$-concave, then the existence of a C-equilibrium is not guaranteed even if it is $\mathscr{L}$-reflective.

Example F Let $N = \{1, 2, 3\}$ and $X = \{a, b, c\}$. The preference profile $(\succsim^i)$, the language $\mathscr{L} = \{\geq_\alpha, \geq_\beta\}$ and the power relation $\trianglerighteq$ are specified as follows:

$\succ^1$	$\succ^2$	$\succ^3$
a	b	a
b	a	c
c	c	b

$\geq_\alpha$	$\geq_\beta$	$\trianglerighteq$
1	2	3
3	3	1
2	1	2

Note that since $1 >_\alpha 3$ and $2 >_\beta 3$ while $3 \triangleright 1$ and $3 \triangleright 2$, weak $\mathscr{L}$-concavity implies

that $3 \rhd 3$. Therefore, $\trianglerighteq$ is not weakly $\mathscr{L}$-concave. This civilized jungle does not have a C-equilibrium. Assume that $\mathbf{x}$ is a C-equilibrium. Agent 1 does not envy agent 2, since otherwise $1 \in J_{\mathscr{L}}(E(\mathbf{x},2))$ (by $\geq_\alpha$) and $1 \rhd 2$. Then, 3 does not envy 2, since otherwise $3 \in J_{\mathscr{L}}(E(\mathbf{x},2))$ (by $\geq_\alpha$) and $3 \rhd 2$. This leaves the assignments $[b,c,a]$ and $[a,b,c]$. The former is not a C-equilibrium since 1 and 2 envy 3 who is not justifiable in N. The latter is not a C-equilibrium since only 3 envies 1, 3 is justifiable in $\{1,3\}$ (by $\geq_\beta$) and $3 \rhd 1$.

4. Existence of a Pareto efficient C-equilibrium

Recall that a Jungle Equilibrium always exists and is Pareto efficient. It follows from Proposition 1 that if the power relation in a civilized jungle with a language of strict orderings is strongly $\mathscr{L}$-concave, then there is a unique C-equilibrium which is Pareto efficient. Thus, the first welfare theorem holds, as it does in a jungle. We will see that this is not the case in a civilized jungle. We start with an example showing that if the power relation is not weakly $\mathscr{L}$-concave (although it is $\mathscr{L}$-reflective), then a C-equilibrium may not be Pareto efficient even if it is unique.

Example G Let $N = \{1,2,3,4\}$ and $X = \{a,b,c,d\}$. The preference profile $(\succsim^i)$, the language $\{\geq_\alpha, \geq_\beta\}$ and the power relation $\trianglerighteq$ are specified as follows:

$\succsim^1$	$\succsim^2$	$\succsim^3$	$\succsim^4$
a	b	a	a
b	a	c	d
c	c	b	b
d	d	d	c

$\geq_\alpha$	$\geq_\beta$	$\trianglerighteq$
2	3	1
4	1	2
1	4	3
3	2	4

Note that since $4 >_\alpha 1$ and $3 >_\beta 1$ while $2 \rhd 4$ and $2 \rhd 3$, weak $\mathscr{L}$-concavity requires that $2 \rhd 1$. Therefore, $\trianglerighteq$ is not weakly $\mathscr{L}$-concave, since $1 \rhd 2$. It is easy to see that $[b,a,c,d]$

is a C-equilibrium, which is Pareto dominated by $[a,b,c,d]$. To see that there is no other C-equilibrium, let $\mathbf{y}$ be a C-equilibrium. Then, it cannot be that $y^1 = a$, since otherwise both 3 and 4 would envy 1 and therefore he would not be justifiable. It cannot be that $y^4 = a$, since then 3 envies 4, is justifiable in $E(\mathbf{y},4)$, and is stronger than 4. If $y^3 = a$, then $y^2 = b$ (otherwise 2 envies 3, is justifiable in $E(\mathbf{y},3)$, and is stronger than 3); however then 1 envies 2, is justifiable in $E(\mathbf{y},2)$ (because 3 is absent), and is stronger than 2. Thus, $y^2 = a$ in a C-equilibrium. It is then straightforward to show that $\mathbf{y} = [b,a,c,d]$.

The observation in Example G is not just a coincidence. The following proposition shows that in a civilized jungle with a language of strict orderings, it is "essentially" true that if the power relation is not weakly $\mathscr{L}$-concave, then we can find a preference profile for which there is no Pareto efficient C-equilibrium. There are exceptions, as shown in Example A, in which the set of C-equilibria is determined by the language independently of the power relation. More precisely, we show that our claim holds unless there is an agent i who is *ranked by* $\trianglerighteq$ *right above* another agent j (i.e. there is no $k \in N$ such that $i \rhd k \rhd j$) who $\mathscr{L}$*-dominates* i in the sense that j is better-suited than i according to every criterion in the language $\mathscr{L}$ (i.e. $j >_\lambda i$ for every $\geq_\lambda \in \mathscr{L}$, denoted by $j \; D_{\mathscr{L}} \; i$).[2]

Proposition 2 *Let $\langle N, X, (\succsim^i), \trianglerighteq \mathscr{L}\rangle$ be a civilized jungle with a language $\mathscr{L}$ of strict orderings such that there are no $i, j \in N$ such that i is ranked right above j by $\trianglerighteq$ and $j \; D_{\mathscr{L}} \; i$. If the power relation is not weakly $\mathscr{L}$-concave, then there is a preference profile $(\succsim^i)$ such that there is no Pareto efficient C-equilibrium.*

[2]It is easy to see that if i is ranked right above j by the power relation $\trianglerighteq$ and j $\mathscr{L}$-dominates i, then the set of C-equilibria of $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$ is identical to that of $\langle N, X, (\succsim^i), \trianglerighteq', \mathscr{L}\rangle$, where $\trianglerighteq'$ is the power relation obtained from $\trianglerighteq$ by swapping the positions of i and j.

Proof. **Step 1:** Suppose that $\trianglerighteq$ is not $\mathscr{L}$-reflective. Then, there exist $i, j \in N$ such that $j \; D_{\mathscr{L}} \; i$ but $i \rhd j$. Assume that among all such pairs, the number of agents who are ranked between i and j according to $\trianglerighteq$ is minimal. Suppose that i is ranked right above agent k by $\trianglerighteq$. We know that $k \neq j$ and $k \not\mathrel{D}_{\mathscr{L}} \; i$. It follows from our choice of i and j that $j \not\mathrel{D}_{\mathscr{L}} \; k$. Next, we construct a preference profile such that there is no Pareto efficient C-equilibrium in the associated civilized jungle. Let $\{\succsim_i, \succsim_j, \succsim_k\}$ be specified as follows:

$\succ_i$	$\succ_j$	$\succ_k$	$\trianglerighteq$
a	a	c	$\vdots$
c	b	a	i
b	c	b	k
$\cdot$	$\cdot$	$\cdot$	j
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Assume that all other agents' most preferred objects are distinct and different from a, b, c. By contradiction, assume that $\mathbf{x}$ is a Pareto efficient C-equilibrium. Since $\mathbf{x}$ is Pareto efficient, $\{i, j, k\}$ must be assigned to $\{a, b, c\}$ while all other agents are assigned to their most preferred objects. If $x^k = a$, then either $x^i = c$ or $x^j = c$, contradicting that $\mathbf{x}$ is Pareto efficient. If $x^i = a$, then i is not justifiable in $E(\mathbf{x}, i)$, since j envies i and $j \; D_{\mathscr{L}} \; i$. This contradicts that $\mathbf{x}$ is a C-equilibrium. Therefore, $[x_i, x_j, x_k]$ must be either $[c, a, b]$ or $[b, a, c]$.

Suppose that $[x_i, x_j, x_k] = [c, a, b]$. Then, $E(\mathbf{x}, j) = \{i, j, k\}$. Next, we argue that k is justifiable in $E(\mathbf{x}, j)$. Since $j \not\mathrel{D}_{\mathscr{L}} \; k$, there exists $\geq_\lambda \in \mathscr{L}$ such that $k >_\lambda j$. Then, since $j \; D_{\mathscr{L}} \; i$, we also have $k >_\lambda i$. Therefore, k is justifiable in $E(\mathbf{x}, j)$ by $\geq_\lambda$ and is stronger than j. This contradicts that $\mathbf{x}$ is a C-equilibrium.

Suppose that $[x_i, x_j, x_k] = [b, a, c]$. Then, $E(\mathbf{x}, k) = \{i, k\}$. Since $k \not\mathrel{D}_{\mathscr{L}} \; i$, we have i is justifiable in $E(\mathbf{x}, k)$. Since i is stronger than k, this contradicts that $\mathbf{x}$ is a C-equilibrium.

Step 2: Suppose that $\trianglerighteq$ is $\mathscr{L}$-reflective but not weakly $\mathscr{L}$-concave. Then, there exist $i, j \in N$ such that for every $\lambda \in \Lambda$, there exists $j_\lambda \in N \setminus \{i\}$ such that $j_\lambda >_\lambda i$ and $j \rhd j_\lambda$, but $i \trianglerighteq j$. Let $I = \{j_\lambda\}_{\lambda\in\Lambda} \cup \{i\}$. Recall that the set $J_{\mathscr{L}}(I)$ consists of agents in I who are the maximizers of $\geq_\lambda$ for some $\lambda \in \Lambda$. Since $\geq_\lambda$ is a strict ordering for every $\lambda \in \Lambda$, we have for each $j \in I \setminus \{i\}$, if $j \notin J_{\mathscr{L}}(I)$ then $J_{\mathscr{L}}(I) = J_{\mathscr{L}}(I \setminus \{j\})$. Let $I^* = \{i, j_1, \ldots, j_m\}$ be a subset of I such that $J_{\mathscr{L}}(I^*) = I^* \setminus \{i\}$. We assume without loss of generality that $j_1 \rhd j_2 \rhd \cdots j_m$.

Let $Z = \{z_0, z_1, \ldots, z_m\}$ be a set of distinct alternatives. Define a preference profile $(\succsim^i)$ such that:

i. Every $j \in I^*$ prefers every alternative in Z to every alternative in $X \setminus Z$ and every $j \in N \setminus I^*$ prefers every alternative in $X \setminus Z$ to every alternative in Z; and

ii. the preferences of the agents in I^* restricted to Z are as follows:

$\succ^i$	$\succ^{j_1}$	$\succ^{j_2}$	$\succ^{j_3}$	$\cdots$	$\succ^{j_m}$
z_0	z_0	z_1	z_2		z_{m-1}
z_m	z_1	z_0	z_0		z_0
$\cdot$	$\cdot$	z_2	z_3		z_m
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$
$\cdot$	$\cdot$	$\cdot$	z_m		$\cdot$
$\cdot$	z_m	z_m	z_1		$\cdot$

Let $\mathbf{x}$ be a Pareto efficient C-equilibrium. Then, by Pareto efficiency, $x^j \in Z$ for every $j \in I^*$. For each $j_k, j_l \in I^*$, if $k < l$, then j_k does not envy j_l. Otherwise, since $j_k \in J_{\mathscr{L}}(I^*)$, we have $j_k \in J_{\mathscr{L}}(E(\mathbf{x}, j_l))$ and $j_k \rhd j_l$, contradicting that $\mathbf{x}$ is a C-equilibrium. Therefore, either i or j_1 must be assigned to z_0 and the agents in $I^* \setminus \{i\}$ must be assigned to objects by running serial dictatorship according to their indices and in ascending order. Thus, we are left with two cases, both of which lead to a contradiction of $\mathbf{x}$ being a C-equilibrium.

Case 1: $[x^i, x^{j_1}, \ldots, x^{j_m}] = [z_0, z_1, z_2, \ldots, z_m]$. Then, $E(\mathbf{x}, i) = I^*$, but $i \notin J_{\mathscr{L}}(I^*)$.

Case 2: $[x^i, x^{j_1}, \ldots, x^{j_m}] = [z_m, z_0, z_1, \ldots, z_{m-1}]$. Then, $E(\mathbf{x}, j_1) = \{i, j_1\}$. Since $i \rhd j_1$ and $\trianglerighteq$ is $\mathscr{L}$-reflective, there exists $\lambda \in \Lambda$ such that $i >_\lambda j_1$. Thus, we have $x^{j_1} = z_0$, even though $i \in J_{\mathscr{L}}(\{i, j_1\})$ and $i \rhd j_1$. □

Thus, Proposition 2 together with (i) of Proposition 1 implies that in a civilized jungle with a language of strict orderings, weak $\mathscr{L}$-concavity of the power relation is essentially necessary and sufficient for the existence of a Pareto efficient C-equilibrium for every preference profile.

5. An analogue to the second welfare theorem

A classical interpretation of the second welfare theorem for an exchange economy is that authorities who are able to assign property rights can induce any Pareto efficient outcome by allocating those rights accordingly. In a jungle, there are no property rights but there is a power relation. In that context, Piccione and Rubinstein (2007) suggested an analogous result: For every Pareto efficient allocation, there is a power relation such that the consequent allocation is an equilibrium of the jungle with that power relation.

We will now prove an analogous result for a civilized jungle. A statement that for a given tuple $\langle N, X, (\succsim^i), \mathscr{L} \rangle$, every Pareto efficient assignment is a C-equilibrium cannot be true (see Example A) since every C-equilibrium $\mathbf{x}$ must be **justifiable** in the sense that every agent j is justifiable in $E(\mathbf{x}, j)$. We say that an assignment $\mathbf{x}$ is **J-constrained efficient** if $\mathbf{x}$ is justifiable and there is no justifiable assignment $\mathbf{y}$ that Pareto dominates $\mathbf{x}$. A J-constrained efficient assignment always exists since we can always find a justifiable assignment by running the serial dictatorship according to a criterion in the language.

We will now show that given a language of strict orderings, for every J-constrained efficient assignment there is a power relation such that the assignment is a C-equilibrium in the civilized jungle with that power relation. Thus, given a language of strict orderings, the authorities can induce any J-constrained efficient assignment by determining the power relation accordingly.

Proposition 3 *Let $\langle N, X, (\succsim^i), \mathscr{L}\rangle$ be a tuple where $\mathscr{L}$ is a language of strict orderings. Then, for every J-constrained efficient assignment $\mathbf{x}$, there exists a power relation $\trianglerighteq$ such that $\mathbf{x}$ is a C-equilibrium for the civilized jungle $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$.*

Proof. Let $\mathbf{x}$ be a J-constrained efficient assignment. Then, for every distinct $i, j \in N$, define $j \; P \; i$ if i envies j and is justifiable in $E(\mathbf{x}, j)$. We first show that the relation P is acyclic. Suppose by contradiction and without loss of generality that $1 \; P \; 2 \; P \cdots \; m \; P \; 1$. For $i = 1$, we identify $i-1$ with m. Define $\mathbf{y}$ by $y^i = x^{i-1}$ for every $i \in I = \{1, \ldots, m\}$ and $y^j = x^j$ for every agent $j \notin I$. The assignment $\mathbf{y}$ Pareto dominates $\mathbf{x}$.

We show that $\mathbf{y}$ is justifiable. Let $i \in N$. If $i \in I$ and j envies i in $\mathbf{y}$, then j envies $i-1$ in $\mathbf{x}$. Therefore, $E(\mathbf{y}, i) \subseteq E(\mathbf{x}, i-1)$. Then, since, by the definition of P, i is justifiable in $E(\mathbf{x}, i-1)$, i is also justifiable in $E(\mathbf{y}, i)$. If $i \notin I$ and j envies i in $\mathbf{y}$, then j also envies i in $\mathbf{x}$. Thus, $E(\mathbf{y}, i) \subseteq E(\mathbf{x}, i)$ and since i is justifiable in $E(\mathbf{x}, i)$, i is also justifiable in $E(\mathbf{y}, i)$.

Finally, let $\trianglerighteq$ be a completion of P. To see that $\mathbf{x}$ is a C-equilibrium for the civilized jungle $\langle N, X, (\succsim^i), \trianglerighteq, \mathscr{L}\rangle$, suppose that an agent i is justifiable in $E(\mathbf{x}, j)$. Then, $j \; P \; i$ and therefore $j \triangleright i$. Agent j is justifiable in $E(\mathbf{x}, j)$ since $\mathbf{x}$ is a J-constrained efficient assignment. Therefore, $\mathbf{x}$ is a C-equilibrium. □

6. Related literature

6.1 Enriching the model with a language

A previous paper that proposes a model with a set of orderings $\mathscr{L}$ (referred to there as *primitive orderings*) is Richter and Rubinstein (2015). Their solution concept, a *primitive equilibrium,* is a pair consisting of a public ordering and a feasible profile of individuals' choices, such that each agent's choice is optimal for him within the alternatives that are not ranked above his choice according to the public ordering. Agents' preferences are required to be $\mathscr{L}$-convex (defined analogously to $\mathscr{L}$-concavity) and the public ordering is required to be a member of $\mathscr{L}$.

As a followup to the current paper, Rubinstein and Yıldız (2021) discuss a model of a society with a language (like the one presented here), but without the addition of a power relation. The proposed equilibrium concept, referred to as *definable equilibrium* (D-equilibrium), is an assignment of the agents to the objects and an attachment of a specific criterion from the language to each object, such that each agent is better-suited, according to the criterion attached to his assigned object, than any agent who envies him. The concepts of D-equilibrium assignment and justifiable assignment are identical, and therefore every C-equilibrium is a D-equilibrium assignment. The key difference is that in a C-equilibrium if there is more than one agent who can justify being assigned to an object by some ordering, then the most powerful agent among them is assigned to the object.

6.2 Relationship to cooperative game theory

A C-equilibrium is an assignment such that every valid objection of the form: "According to a legitimate criterion, I am the best-suited agent from among the group of agents who wish to be assigned to the object" can be responded to by the agent assigned to the object using a counter-objection of the form: "According to a different legitimate criterion, I am also the best-suited agent in that same group, and furthermore I am stronger than you." As such, the structure of the C-equilibrium concept is similar to that of many solution concepts in cooperative game theory, according to which an outcome is a solution if for any valid objection (by some definition), there is a valid counter-objection (by some definition). This structure can clearly be discerned in both von Neumann and Morgenstern (1953)'s *stable set* and Aumann and Maschler (1961)'s *bargaining set* and is also consistent with most cooperative solution concepts, including the *Nash Bargaining Solution*, as discussed by Rubinstein, Safra, and Thomson (1992). The objections and counter-objections in our model depend on the set of criteria used to justify the assignment of an agent to an object, whereas in the case of cooperative solution concepts the justification typically involves an action by a coalition that includes the objector or the counter-objector. Piccione and Razin (2009) apply such a cooperative game-theoretic approach to a jungle model in which agents have identical preferences and the power relation is over coalitions rather than individuals.

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