Modeling Bounded Rationality in Economic Theory: Four Examples *

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ABSTRACT: The paper provides an introduction to the growing literature in Economics on models of Bounded Rationality, in which reasoning procedures are the cornerstones of the model. Four models are presented, each of which demonstrates a different type of Bounded Rationality:

(i) Limited ability to solve a set of propositions: A mechanism design problem is investigated in an environment where agents try to cheat effectively using a procedure which is anchored in the truth.

(ii) Reducing the complexity of strategies: Each player in a Repeated Prisoner's Dilemma desires not only to increase his payoff but also to reduce the complexity of his strategy.

(iii) Belief formation on the basis of a small sample: A model of manipulative voting is presented in which a voter forms beliefs about his vote being pivotal based on a small sample of observations of other voters' intentions.

(iv) Diversified views of the world: A seller exploits differences in buyers' abilities to recognize time series patterns in order to increase his profits beyond what they would be with a fixed price.

KEYWORDS: Bounded Rationality, Economic Theory, Procedures of choice.

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1. Introduction

The terms *Bounded Rationality* and *Economic Theory* mean different things to different people. For me (see Rubinstein (2012)), Economic Theory is a collection of stories, usually expressed in formal language, about human interactions that involve joint and conflicting interests. Economic Theory is not meant to provide predictions of the future. At most, it can clarify concepts and provide non-exclusive explanations of economic phenomena. In many respects, a model in Economic Theory is no different than a story. Both a story and a model are linked to reality in an associative manner. Both the story teller and the economic theorist have in mind a real-life situation but do not consider the story or the model to be a full description of reality. Both leave it to the reader to draw their own conclusions, if any.

Models of Bounded Rationality are for me (see Rubinstein (1998)) models that include explicit references to procedural aspects of decision making, which are crucial for the derivation of the analytical results. A common critique of bounded rationality models is that they are more specific, less general and more arbitrary than models in the mainstream areas of Economic Theory, such as general equilibrium or game theory in which full rationality is assumed. In response, I would claim that every model makes very (very) special assumptions. Without strong assumptions, there would be no conclusions. It is true that rationality is a special assumption since it is viewed by many as being normative whereas models of bounded rationality are viewed as dealing with deviations from normative behavior. Returning to the analogy of a story: What story is more interesting - one about normative people who behave "according to the book" or one about people who deviate from normative behavior?

Not every model that is inconsistent with some aspect of rationality is a model of bounded rationality. A model in which rational agents ignore some aspect of rationality is a bad model rather than a model of bounded rationality. A good model of bounded rationality should include a procedure of reasoning that "makes sense" and is somewhat related to what we observe in real life.

I am not a big fan of abstract methodological discussions. I prefer to demonstrate an approach by discussing examples. Accordingly, the article discusses four models, in which economic agents are assumed to reason in systematic ways using a well-defined procedure that is outside the standard scope of rational behavior. My choice of models is totally subjective. I am partial to these models because I was involved in constructing and analysing them over the last 35 years. Rubinstein (1998) surveyed the field at the time, while Spiegler (2011) surveys more recent models in which aspects of bounded rationality have been inserted into classic models of Industrial Economics.

2. Bounded Rationality and Mechanism Design

Story: The director of a prestigious MBA program has been persuaded by Choi, Kariv, Müller, and Silverman (2014) that transitivity is strongly correlated with success in life (as measured by wealth). Thus, he decides to accept only those candidates who hold transitive preferences.

Accordingly, the director designs a simple test. He presents three alternatives a, b and c to each candidate and asks them to respond to a questionnaire consisting of three questions $Q(a, b)$, $Q(b, c)$ and $Q(a, c)$ where $Q(x, y)$ is the quiz question:

Do you prefer x to y or y to x?

◯ I prefer x to y.

◯ I prefer y to x.

For each of the three questions a candidate must respond by clicking on one and only one of the two possible answers. Thus, the questionnaire has eight possible sets of responses.

The director is obligated by law to inform candidates about the conditions that will gain them admission to the program. The director has specified the following two conditions:

R1: If you prefer a to b and b to c then you must prefer a to c.
R2: If you prefer c to b and b to a then you must prefer c to a.

The candidates are reminded that an "if" proposition is violated only if its antecedent (the "if" part) is satisfied and its consequent (the "then" part) is not.

The director was hoping that the candidates would feel obliged to report the truth and thus the simple questionnaire should separate perfectly between "good" candidates who hold transitive preferences and "bad" candidates who do not. In order to encourage instinctive responses, the director also sets a short time limit for completing the questionnaire.

A disappointment: the director learns that all the candidates have been admitted and concludes that the candidates with cyclical preferences had gamed the system. The frustrated director opens an investigation. Researchers are rushed to the scene. They interview candidates to reveal how they answered the questionnaire so "successfully". Apparently, candidates treated the questionnaire as a puzzle to be solved in order to be admitted to the program. The time limit made solving the "puzzle" a challenging task. The following procedure was identified:

Step 1: Examine whether your honest set of answers satisfies all the conditions.

If it does, then happily submit those answers. If not, go to step 2.

Step 2: Find a condition that is violated by your honest answers (that is, your true answers satisfy the antecedent but not the consequent of the condition). Try modifying your answers with respect only to the consequent. If the modified set of answers satisfies all conditions, then submit them. If not, iterate step 2 (starting with your honest set of answers) until it is exhausted. Then, proceed to Step 3.

Step 3: Give up. Submit the honest answers (and be rejected).

Faced with the questionnaire and the admission conditions, a "good" candidate (who has transitive preferences) reports the truth and is admitted. What about a "bad" candidate with cyclical preferences $a \succ b \succ c \succ a$ (or $c \succ b \succ a \succ c$)? An honest response satisfies R2 (the antecedent is false) but violates R1. The candidate remains with the true answers to $Q(a,b)$ and $Q(b,c)$ and modifies his answer to $Q(a,c)$. Thus, he responds as if he holds the preferences $a \succ b \succ c$ and is admitted. In this way, all the candidates are admitted.

Realizing that people are prepared to cheat (when they have to) by using the procedure to find a persuasive set of responses, the director tries to come up with a modified set of admission conditions so that all the good candidates will pass the test while the bad candidates will not. He adds two new admission conditions, so that the new set of admission conditions is as follows:

R1: If you prefer a to b and b to c, then you must prefer a to c.
R2: If you prefer c to b and b to a, then you must prefer c to a.
R3: If you prefer a to b and a to c, then you must prefer c to b.
R4: If you prefer c to a and c to b, then you must prefer a to b.

This set of conditions seems a bit odd. How would the candidates respond to it?

First consider a "good" candidate who holds the transitive relation $b \succ a \succ c$ (or one of the following three other preferences : $b \succ c \succ a$, $a \succ c \succ b$ or $c \succ a \succ b$). None of the antecedents of the four conditions are satisfied and responding truthfully leads to acceptance, as desired by both the candidates and the manager.

What about a "good" candidate who holds the preferences $a \succ b \succ c$ (or $c \succ b \succ a$)? If he responds truthfully then he violates R3 and is rejected. However, he is induced by the violation of R3 to modify his answer to $Q(b,c)$; this leads him to respond as if he holds the preferences $a \succ c \succ b$ and thus he cheats successfully.

On the other hand, a "bad" candidate with cyclical preferences $a \succ b \succ c \succ a$, (or their counterpart) violates only R1 and is led to try only the set of answers corresponding to the preferences $a \succ b \succ c$ which do not satisfy the set of conditions. Thus, such a candidate fails the test, as desired by the program director.

A happy ending, at least for the program director. Interestingly, he does not mind that some candidates cheat. The questionnaire "works" in the sense that it separates between the good and bad candidates. Actually, the director also finds that there is no alternative set of conditions which will separate between the two types of candidates without some good candidates having to cheat. He concludes that "white lies" are sometimes necessary.

Discussion: This section is based on Glazer and Rubinstein (2012). The model comes under the rubric of Bounded Rationality since it explicitly specifies a process of reasoning used by the individuals. The paper characterizes the circumstances under which the designer of the mechanism is able to implement his target given that the individuals use the discussed procedure. Conditions under which implementation does not require that some "good" candidates need to cheat are established as well.

Noone claims that most people use this exact procedure. Nonetheless, Glazer and Rubinstein (2012) provide experimental evidence that the two central ingredients of the procedure are present in many people's minds: (i) The truth is the anchor for cheating. When one invents a false set of answers, he starts from the truth and modifies it to look better. (ii) Given that an "if" sentence is violated, an individual who wishes to cheat successfully will modify his true set of answers by reversing his answer to fit in the *consequent* of the condition. Note that without (i) separation between the candidates would be impossible.

In the current story, the bounded rationality of the candidates is an advantage for the designer. If all candidates were fully rational, as is commonly assumed in the mechanism design literature, all of them would be able to game the system if necessary. It is the cognitive imperfection of the individuals which *opens a door* to obtaining a desirable outcome.

Bibliographic comments:

(i) Glazer and Rubinstein (2014) study a related model in which a candidate responds to a questionnaire without being notified of the acceptance conditions though he has access to the data about the set of responses that achieve admission and tries to make sense of things. There is a bound on the complexity of the regularities that the candidates can detect in the data. It is shown that whatever this bound is, the director can construct a sufficiently complex questionnaire such that agents who respond honestly to the questionnaire will be treated optimally and the probability that a dishonest agent will cheat successfully is "very small".

(ii) de Clippel (2014) investigates Maskin's classical implementation question in environments in which individuals follow systematic procedures of choice that are not consistent with rational behavior.

(iii) Li (2017) is motivated by an observation that bidders in an ascending bid auction tend to wait until the bid reaches their reservation value whereas a majority of bidders don't report the true value in the "equivalent" second-bid auction. An *obviously dominant strategy* is one in which the decision at each decision node is "obvious" in the sense that it does not depend on conjecture about the other players' actions. Li identifies several targets that are implementable using an extensive game in which the desirable outcome is obtained through obviously dominant strategies. Glazer and Rubinstein (1996) is an earlier paper that suggests a different criterion for simple implementation.

3. Elections and Sampling Equilibrium

Story: Three candidates are running for office. A large number of voters will participate in the elections and each will cast one vote in favor of one of the candidates. The issues on the agenda are "hot" and all of the voters are expected to cast their ballots. The winner will be the candidate with the largest number of votes (even if he does not achieve an absolute majority).

The candidates are labeled as Left, Center and Right. The population of voters is split into three classes *Leftist*, *Centrist* and *Rightist* with the proportions λ, μ and ρ:

Leftist: Individuals with the preferences $L \succ C \succ R$.

Centrist: Individuals with the preferences $C \succ L \sim R$.

Rightist: Individuals with the preferences $R \succ C \succ L$.

It is known that the Leftist and Rightist groups are more or less of equal size and that the Centrist group is smaller but not negligible $(11/3 < \lambda = \rho < 3/7)$. Some observers predict that C will win since some of the leftists and rightists will vote for C in order to prevent the election of the candidate on the other extreme. Some view such a result as a desirable compromise. Others find it unacceptable that the least popular candidate might win the election.

Voters by nature tend to vote sincerely, but they will nonetheless vote for another candidate if they think that their candidate will certainly lose and the race between the other two candidates is close. In such an event, they vote for their second choice.

Surveys are forbidden and voters base their prediction of the election results on occasional discussions with casual acquaintances who express their voting intentions (without specifying their first choice). As is often the case, people say that they "talked with several people" but actually mean that they talked with just two. Thus, based on a sample of size two, people decide who to vote for. Therefore, they may change their mind during the campaign according to the sample results.

Some game theorists and political scientists analyse complicated models in which voters think strategically, in the sense that they put themselves in the shoes of other voters, calculate correctly the distribution over states conditional on the infinitesimal probability event that their personal vote will be pivotal and vote optimally given that event. No traces of such behavior can be found in our story (nor in real life). No standard game-theoretical considerations are used. At each moment in the campaign, each voter constructs his prediction and decides who to vote for based on the small sample he has drawn.

Accordingly, if a voter samples two people who intend to vote for the same candidate the voter treats the election as already decided, concludes that his vote will not make any difference and votes sincerely. If the two sampled individuals intend to vote for two different candidates, he views himself as pivotal and votes for the candidate he prefers out of these two (who is not necessarily his favorite). Thus, a C supporter intends to vote sincerely regardless of the sample findings. An L supporter will vote sincerely unless he draws a sample of "C and R" and then votes for C. An R supporter will vote for C if his sample is "C and L" and sincerely otherwise.

Thus, although voters are stubborn in their political views and never change their basic position, they may change their intention during the campaign on the basis of the sample. The proportions of voters voting for each candidate will fluctuate until eventually stabilizing around a distribution of votes which we call *equilibrium*.

Given that all C followers vote C, L supporters will never vote R and R supporters will never vote L, a distribution of votes is characterized by two numbers: a, the proportion of the L camp that votes C, and b, the proportion of the R camp that votes C. In equilibrium, those proportions are stable. Notice that even when equilibrium is reached members of the L and R camps may still change their intentions; however, in equilibrium, the changes are "balanced" and the distribution of votes remains constant.

Election day arrives. The ballots are closed. Will C win? Let's examine the equilibrium. Recall that the distribution of the L,C,R groups is $(\lambda, 1-2\lambda, \lambda)$. If at some point in time the proportions of manipulative voters are a and b, then the proportions of voters' intentions (following the above procedure) is $(\lambda(1-a), \lambda a+(1-2\lambda)+\lambda b, \rho(1-b))$. The proportion of L supporters who will sample "C and R" is $2[\lambda a+(1-2\lambda)+\lambda b][\lambda(1-b)]$. The distribution is stable if this proportion is equal to a (and analogously for R). Thus, equilibrium is characterized by two equations:

$a = 2[\lambda a+(1-2\lambda)+\lambda\, b][\lambda(1-b)]$ and $b = 2[\lambda a+(1-2\lambda)+\rho b][\lambda(1-a))]$.

This set of two equations has a unique solution which can easily be calculated and accordingly the minority candidate C will win! Incidentally, when the size of C's base drops below 1/7, candidate C would lose, at least according to the story we have told.

Discussion: This section is based on Osborne and Rubinstein (2003). The bounded rationality component of the model is the procedure used by each voter to estimate the results. The paper analyses some variants of the model. An example of a result: R will not be elected whenever his base is smaller than that of L, regardless of C's base.

As mentioned before, the procedure of choice assumed here is in sharp contrast to what is assumed in standard game-theoretical voting models, in which a voter is assumed to fully calculate the conditions under which his vote will be pivotal given the correct expectations about the votes of all other voters.

Bibliographic comments:

(i) The model was inspired by Osborne and Rubinstein (1998) who proposed the concept of S-1 equilibrium and applied it to symmetric two-player games. A population of players is pair-wise randomly matched to play a game. A newcomer to the population samples each of the potential actions once by interviewing one person whose experience depends on the behavior of the randomky matched player he played against. Then, the newcomer chooses the action which, according to the sample, yields the best outcome. An equilibrium is a stable distribution of choices in a population, in the sense that the probability that a player chooses a particular strategy is equal to the frequency of the strategy in the distribution. This equilibrium concept differs fundamentally from the standard game-theoretical analysis and has special properties. For example, a dominated strategy (which is never the best response against any belief about the other player's behavior) might appear with positive probability in the support of the equilibrium when the symmetric game has at least three strategies.

(ii) Rani Spiegler constructed several economic models in which individuals follow an S-1 type of procedure (see Spiegler (2011) (chapters 6 and 7)). Spiegler (2006a) presents and analyses a model of a market for quacks. The market consists of several healers whose success rates are identical to that of "non-treatment". The healers compete on price. Patients rely on "anecdotal" evidence regarding the healers' success and also sample the free non-treatment. Spiegler (2006b) presents a model of competition between providers of a service in which each chooses a distribution of "price" in the range $(-\infty, 1)$. Consumers sample each of the providers and choose the one with the lowest price (which might be negative). The uniqueness of the symmetric Nash equilibrium with expected price of 1/2 is proven. Increasing the number of competitors leads to a mean-preserving spread in the equilibrium price distribution.

4. Long Interactions and Finite Automata

Story: Two players are involved in a repeated Prisoner's Dilemma interaction. Each of the players chooses one of two modes of behavior each day: C (cooperative) or D (non-cooperative). A player's daily payoff if he chooses the action corresponding to a row while his partner chooses the action associated with a column is given by the corresponding entry in the following matrix:

	C	D
C	3	1
D	4	0

Each period ends with one of four outcomes $(C,C),(C,D),(D,C)$ or, (D,D) and the four pairs of payoffs that can be obtained each day are $(3,3)$, $(1,4),(4,1)$ or, $(1,1)$. The infinitely long interaction yields an infinite sequence of outcomes and two streams of payoffs, one for each player. A player cares only about his long-term average payoff.

In the long-term interaction (i.e. a repeated game), each player chooses a strategy, i.e. a plan that specifies whether to play C or D after every possible sequence of his opponent's actions. A strategy can be complex since it specifies an action for an infinite number of contingencies. A player's "language" in formulating a strategy is a *finite automaton* (machine). This is not an actual machine used to play the strategy but rather an abstract description of the mental process used by players to determine an action at each point in time after any history he might encounter.

A finite automaton consists of :

(i) a finite set of states (of mind);

(ii) an initial state from which the machine starts operating;

(iii) an output function that indicates, for each state, one action (C or D) which the player will play whenever his machine reaches that state.

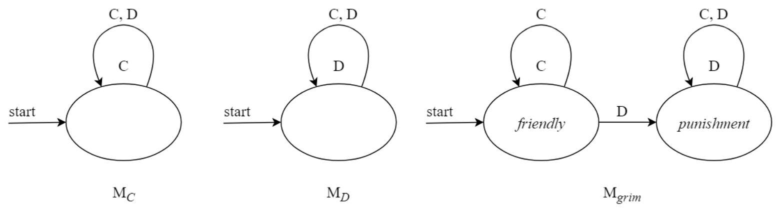
(iv) a transition function which determines the next state of the machine for each state and each possible observation (C or D) of the action taken by the other player.

The two machines are operated in the expected way: The two initial states and the two output functions determine the first-period pair of actions. Each player observes the other player's action and his transition function determines the next state of his machine. The process continues recursively as if each pair of states is the initial one.

Any weighted average of the four one-shot payoff pairs that equal at least 1 (a payoff each player can guarantee by playing D) is a long-term average payoff vector of some stable pair of strategies (stable in the Nash equilibrium sense). In other words, no player can increase his average payoff by using a different strategy. To obtain such a pair of average payoffs (π_1, π_2), players can play in a cycle of length $n_{D,D}+n_{D,C}+n_{C,D}+n_{C,C}$ so that the outcome (i, j) will appear $n_{i,j}$ times in the cycle such that $\frac{n_{1,1}(1,1)+n_{4,0}(4,0)+n_{0,4}(0,4)+n_{3,3}(3,3)}{n_{1,1}+n_{4,0}+n_{0,4}+n_{3,3}} = (\pi_1, \pi_2)$. Players will support this path of play by threatening that any deviation will trigger moving to the non-cooperative behavior forever.

However, in our story and unlike the standard repeated game model, a player cares not only about his stream of payoffs but also about the complexity of the machine he employs, which is measured by the number of states in the machine (without taking into account the complexity of the transition function). Players view the complexity as a secondary consideration after the desire to increase the average payoff. Thus, a pair of machines will not be stable if one of the players can replace his machine with another that either yields a higher average payoff or the same payoff but less complexity. In other words, each player has lexicographic preferences with the first priority being to increase average payoff and the second to reduce complexity.

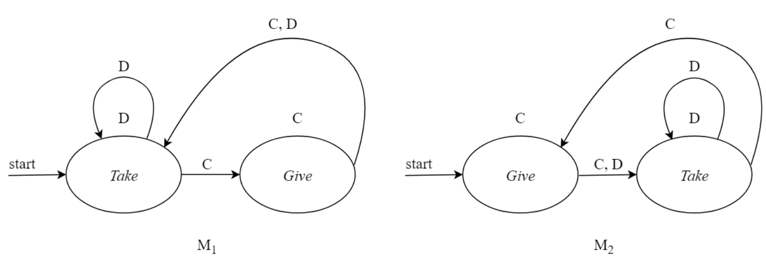
Initially, the two players use a naive strategy - the one-state machine M_C (see Figure 1) which plays C independently of what is observed. This situation is not stable. One of the players modifies his machine to M_D, which plays D independently of what it observes. Such a deviation improves his payoff each period without increasing complexity.



The two players then adopt the machine M_{grim}. This machine has two states : *friendly* and *punishment*. The initial state is *friendly*. The output function assigns C to *friendly* and D to *punishment*. In the case that D is observed when the machine is at *friendly* it moves to *punishment,* which is a terminal state. A player who plays against M_{grim} cannot increase his long-term average payoff whatever machine he chooses. However, eventually one of the players notices that he can reduce the complexity of his machine without losing payoff by dropping the state *punishment* which does not lead to any loss in terms of payoff.

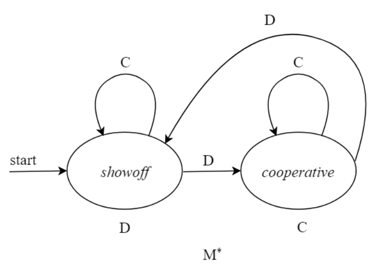
The situation has some happier endings. Following are two examples:

Give and Take : Player 1 chooses the two-state machine M_1 (see Figure 2) with initial state *Take*, in which he plays D and moves to the second state *Give* after the other player plays C. If M_1 reaches *Give* it plays C and moves to *Take* independently of what it observes. Note that "giving" means playing C and tolerating the other player playing D. "Taking" means playing D and expecting the other player to play C. Player 2 adopts the machine M_2 which is identical to M_1 except that its initial state is *Take*.



Adopting this pair of machines results in stability. Players expect to alternate between giving and taking. A player's threat against the other player "not giving" is to avoid moving to *Give,* a state in which he is supposed to give until the other player gives. As long as player 1 follows M_1 and player 2 follows M_2, the players will alternate turns between giving and taking and each player obtains an average payoff of 2.5. No player can save on states without losing on payoff and no player can increase his average payoff no matter what he does.

The cooperative equilibrium: Each player uses the machine M^* (see Figure 3) which has two states: *Showoff* and *Cooperative*. In *Showoff*, a player plays D and moves to *Cooperative* only if the other player plays D. In *Cooperative,* the machine plays C and moves to *Showoff* only if the other player played D. The machine's initial state is *Showoff*. Thus, players start by proving their ability to punish one another and then and only then move to the cooperative mode.

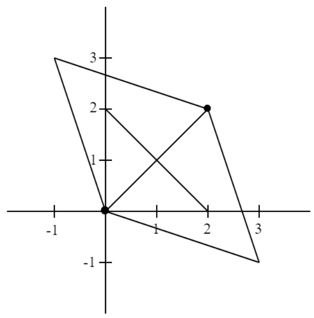


Each player obtains an average payoff of 3. A player cannot increase his payoff whatever he does. Reducing the number of states would cause a payoff loss (the one-state machines M_D and M_C achieve an average of only 1 or 0 against M^*).

Discussion: This section is based on Rubinstein (1986) and Abreu and Rubinstein (1988). The bounded rationality component of the model is the complexity of the strategy and the desire of players to minimize it as long as the repeated game payoff is not reduced.

The two equilibria described above demonstrate the logic of two different types of social arrangements. In the first, players alternate between giving and taking; the threat not to give tomorrow deters the other player from not giving today. The second arrangement achieves ideal cooperation but on the way players must demonstrate their ability to punish if necessary.

Abreu and Rubinstein (1988) characterize the Nash equilibria of the infinitely repeated game with finite automata for (i) general one-shot two-player games; (ii) discounting evaluation criterion; and (iii) any preferences that are increasing in the payoffs and decreasing in complexity. The payoff vectors that can be obtained are those on the "cross" in figure 4:



A few structural results were proven, such as that the two machines must have the same number of states in equilibrium and that there is a one-to-one correspondence between the occurrence of states in the two machines (namely, in equilibrium a machine "knows" the state of the other player's machine). An improved presentation of the results, following Piccione (1992), appears in Rubinstein (1998, chapter 8).

In a previous paper, Rubinstein (1986) showed that the only equilibria of the machine game with the additional constraint that all states are revisited infinitely often (otherwise they would eventually be dropped) yield either the non-cooperative outcome or cyclical combinations of (C,D) and (D,C) only. Cooperation cannot be obtained in such an equilibrium.

Bibliographic comments:

(i) Neyman (1985) and Ben Porath (1993) investigate a repeated game with finite automata in which the number of states a player can use in his machine is exogenously bounded. Neyman (1985) shows that in the finite-horizon Prisoner's Dilemma, cooperation can be achieved in equilibrium by applying the idea that the machines waste their limited resources by following an initial string of actions ("a password to heaven") that prevents them from identifying the point in time - towards the end of the game - at which it is profitable for them to deviate to the non-cooperative mode of behavior without being punished in the future. Ben Porath (1993) shows that in an infinitely repeated zero-sum game, if players use the "limit of the means" criterion and are limited in the size of the automaton they can use, then as long as the number of states of one machine is not much larger (in a well-defined sense explained there), the equilibrium payoff vector will be close to the equilibrium payoff of the basic game.

(ii) Examples of other models of repeated games with bounds on the strategies: Lehrer (1988) studies the model of repeated games with bounded recall and Megiddo and Wigderson (1986) and Chen, Tang and Wang (2017) study the model of repeated games in which players use Turing machines instead of finite automata.

(iii) Eliaz (2003) and Spiegler (2004) study models of repeated games and bargaining where the simplicity considerations were applied to the player's belief regarding the opponent's strategy, rather than his own.

5. Agents with different models in mind

Story: Each of two agents, 1 and 2, wish to purchase a particular service each day. The service is provided by P (a provider) but can also be obtained from a local source at a price of 6 by agent 1 and at the price of 4 by agent 2.

All involved parties interact daily for a "very long period of time". Every morning, without knowing the price charged by P, each agent decides whether to purchase the service from the local source or to go to P. A cost of 3 is associated with each daily trip to P.

While the agents are making their daily decisions, P posts a price at which he commits to sell the service to whoever asks for it. The provider P can supply the service at no cost to one or both agents. An agent who comes to P is informed of the price offer and decides whether to buy the service from P or return to the local price. At the end of the period, the price becomes common knowledge to all individuals.

The provider cares only about his average long-run profits. He commits to a sequence of prices. An agent behaves myopically in every period. He makes the trip to P only if his expected costs (given his beliefs) do not exceed the local charge. He bases his belief on the regularities he detects in this sequence. The ability to detect a regularity is given by a non-negative integer k (referred to as the *order*) which expresses a player's "cognitive depth": an agent of order k detects all regularities of the type "after a particular string of k prices the price will be..." but cannot detect more complicated regularities. Thus, an agent of order 0, for example, only learns the frequency at which different prices appear in the price sequence. An agent of order 1 learns the frequency of each price given the last price. If the real sequence is $(0,0,1,1,0,0,1,1....)$ an agent of order 0 or 1 will always believe that the next element in the sequence is 0 or 1 with equal probability. An agent of order 2 (or higher) will accurately predict the next element in the sequence since the last two periods fully "encode" the next element.

Our two agents differ in their ability to recognize patterns. While agent 1 can detect a pattern that determines the next price based on the last three periods ($k_1 = 3$), agent 2 is more sophisticated (which aligns with the assumption that his outside option is better than agent 1's) and can detect patterns on the basis of the last 4 periods ($k_2 = 4$). Nothing essential would change in the story if we replace these bounds with any other non-negative integers k_1 and k_2, as long as $k_1 < k_2$.

The provider understands that if he always charges a price higher than 3 then he will not attract any customers; if he constantly charges a price between 1 and 3, then only agent 1 will make the trip; and if he always charges a price of 1 or below, then both agents will buy his service but his profits will be at most 2. Thus, by charging any constant price P his daily profits cannot exceed 3.

Furthermore, the provider realizes that if both agents have the same cognitive limit, then using random price sequences will not help him since both agents will eventually compare their local price to the expected price plus 3 and he would not make more than what he would have with a constant price.

Finally, it comes to the provider's attention that he can obtain higher profits if he wisely exploits the differences in the agents' cognitive ability and the correlation between an agent's order and the agent's outside option, namely that the more sophisticated agent has a better outside option.

He learns about the existence of "DeBruijn sequences of order k" (for every k). To understand this concept, consider the following infinite sequence of prices which has a cycle of length 16 and is an example of a a DeBruijn sequence of order 4. :

$(5, 5, 5, 5, 1, 5, 5, 1, 1, 5, 1, 5, 1, 1, 1, 1, ...)$.

An agent with $k = 16$ would, of course, be able to detect a rule that creates this sequence and to correctly predict the next period's price. In fact any agent with $k \geq 4$ would also be able to. The sequence is built so that all 16 combinations of 4 prices (1 and 5) appear in the cycle exactly once and thus the last four prices predicts the next element in the sequence. On the other hand, the last three values in the sequence predict that it is equally likely that the next price will be 1 or 5. Thus, an agent with $k = 4$ would be able to predict that a low price follows four high prices, that after a sequence of three high prices and one low price always comes a high price, etc. An agent with $k = 3$ will always maintain the belief that the next price will be either 1 or 5.

Thus, Agent 2 will know when the low price will be posted and will approach the provider only on mornings when he expects the price to be 1. Agent 1 is not able to find any useful regularity. After the appearance of any 3 numbers, he will believe that there is an equal chance of observing a high or a low price. His expected price is 3, so he every morning he approaches the provider and buys the service.

Thus, the provider has found a non-constant sequence, which is complicated enough that agent 1 is left confused and regular enough so that agent 2 is able to predict fairly well. The provider's expected profit is 7/2 per period, which is more than he could obtain using any constant price sequence.

Discussion: This section is based on Piccione and Rubinstein (2003). The bounded rationality element in this model is the limited ability of agents to recognize patterns. Once there is a correlation between this ability and other economic factors, sophisticated manipulators can try to use the correlation as a way to sort out agents to their benefit. Conditions for this manipulation are analysed in the paper.

From an economic perspective, this line of research is related to Spence (1973). However, whereas in Spence (1973) the correlation is between two materialistic factors (productivity and ease of performing a worthless task), here the separation between agents is based on the correlation between a materialistic factor (the willingness to pay) and cognitive ability (to recognize patterns in a time series). Note that the separation can also emerge in market equilibrium without the explicit interference of an interested party.

Bibiliographic comments:

(i) Rubinstein (1993) is an earlier paper demonstrating the ability of a monopolist to use the correlation between cognitive differences and willingness to pay in order to increase his profits. The modeling of agents' limits in understanding a multi-part price offer makes use of the formal concept of a perceptron.

(ii) Eyster and Piccione (2013) is a model of competitive markets, in which risk-neutral traders trade a one-period bond against an infinitely lived asset in each period. Traders lack structural knowledge of the situation and use various incomplete theories of the type discussed in this section in order to form statistically correct beliefs about the long-term asset price in the next period. One of the results is that the price of the long-term asset is affected by the diversity in the agents' cognitive levels.

(iii) In a series of papers starting with Jehiel (2005), Philippe Jehiel developed the concept of *analogy-based equilibrium*. The basic idea is related to the way we play games like chess. When planning a move, we evaluate the board positions that it can lead to. In analogy-based equilibrium, those evaluations are "correct on average". Players differ in their partition of the set of situations they bundle together, where finer partitions reflect a better understanding of the situation.

(iv) Spiegler (2011, ch 8.) includes a pedagogical exposition of the model and links it to other concepts like Jehiel's.

(v) Rani Spiegler suggests a general framework for studying economic agents who have imperfect understanding of correlation structures and causal relations (Spiegler (2016)). Spiegler (2018) identifies a condition under which the wrong model does not allow an outsider to systematically fool the agent.

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