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Irrational diversification in multiple decision problems

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Abstract

The paper deals with multiple decision problems, which are *similar* to the task of guessing the color outcomes of five independent spinnings of a roulette wheel, 60% of whose slots are red and 40% white. Each correct guess yields a prize of $1. The guess of 5 Reds clearly first order stochastic dominates any other strategy. In contrast, subjects diversify their choices when facing a multiple decision problem in which the choice is between lotteries with clear objective probabilities. The diversification is stronger when the subjects face uncertainty without objective probabilities and weaker when the choice problem involves real life actions.

JEL classification: C9

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1. Introduction

The starting point of this project was an observation made by Amos Tversky and myself during the analysis of experiments we had conducted for a very different project (see Rubinstein et al., 1996). During the pilot stage of

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that research, we presented the subjects with sequences of similar games. In each game, a subject had to guess the site where an opponent had hidden a prize. The labels of the sites were arranged in a row. The problems differed only in the labels given to the various sites (e.g., A,B,A,A; 1,2,3,4; or three smileys and one sad face). We observed that subjects employed rules to play the game (such as avoiding edges, choosing the salient label or avoiding the salient label) but we got the impression that subjects diversified the rules they used during the sequence of games. Thus, after employing a certain rule in, for example, three games, they switched to a different rule in the fourth. This phenomenon introduced a significant amount of noise into our data. There was no clear rationale for this diversification. We concluded that it was an expression of a more general phenomenon in which people tend to diversify their choices when they face a sequence of similar decision problems and are uncertain about the right action. This observation prompted us in 1993 to investigate the issue of diversification experimentally. Amos Tversky died in 1996 and for a long time I was reluctant to continue our joint research. Though all the experiments reported here were conducted in 1998/9, the ideas originated in my joint work with Amos.

The paper deals with decision problems, which are *similar* to the task of guessing the color outcomes of five independent spinnings of a roulette wheel, 60% of whose slots are red and 40% white. Each correct guess yields a prize of $1. The guess of 5 Reds clearly first order stochastic dominates any other strategy. (Note that if a decision maker chooses white for, let us say, the third outcome, he simply increases the probability that he will get the third dollar from 0.4 to 0.6 if he changes his choice to red.)

In all the experiments reported in this paper the subject's task was to make five choices, each of which was a choice element in a fixed set of alternatives. The consequence of each of the five choices was either 'success' or 'failure' according to the realization of a random factor. The best strategy as determined by the rewards and the structure of the random factor was to choose five times that action which is most likely to achieve success.

The results confirm the hypothesis that subjects diversify their choices when facing a multiple decision problem in which the choice is between lotteries with clear objective probabilities. The diversification is stronger when the subjects face uncertainty without objective probabilities and weaker when the choice problem involves real life actions. The results are quite strong considering the fact that all the subjects in this study were either undergraduate or graduate students of economics who had completed at least a basic course in statistics and the majority had taken a course in microeconomics or even game theory.

2. Methods

As stated, the experimental work for this paper was done with economics students at Tel Aviv University. Some of the experiments were conducted via the Internet as part of the assignments in an introductory course on game theory (see Rubinstein (1999) for a description and assessment of that method). The majority of surveys were conducted through forms filled by the students in class (the form was actually sandwiched between forms for two other unrelated experiments).

No monetary rewards were offered. I am fully aware (see Rubinstein, 2001) of the reluctance of many experimentalists not to offer monetary rewards. I am also aware of the fact that classroom experiments may not be taken seriously by some of the subjects (the same may apply to real life choices...). However, I do not believe that adding a prize of a few dollars would induce students to take the situation much more seriously. It is my view that the conventional experimental methods cannot provide a meaningful quantitative assessment of real-life behavior and in any case can only indicate the existence of certain patterns of behavior.

3. Strong diversification in lottery-type decision problems

A strong tendency to diversify was observed in multiple decision problems in which the prospects of making the right decision were specified in terms of objective probability and the choice was clearly framed as being a choice between 'lotteries'.

Subjects were asked to answer the following question (in Hebrew):

Problem 1 ('*Guess the colors*')**.** You are participating in the following game: Five cards are chosen randomly from a deck of 100 cards. The deck is composed of colored cards according to the following breakdown: 36 of them Green, 25 Blue, 22 Yellow and 17 Brown. The five cards are then placed into five separate envelopes marked A, B, C, D and E. You have to guess the color of the card in each envelope (by marking an X in the matrix below). Imagine that you will receive a prize for each correct guess (you can receive up to five prizes).

(The subject is then presented with a 5×4 matrix where each row corresponds to one guess).

In two separate undergraduate classes (numbering 50 and 74 students), only 42% and 38% of the students, respectively, chose five Greens. A popular alternative strategy is that of 'probability matching', in which a subject diversifies by choosing a mixture of actions in proportions similar to the

probabilities of success (in our case, a choice of 2 Greens, 1 Blue, 1 Yellow and 1 Brown). About 30% of the subjects (52% of those who diversified) chose the '*probability matching*' strategy.

I also conducted a modified version of Problem 1 experiment in which the subjects were told that one of the five envelopes had already been selected and he could win only one prize if and only if his guess regarding that envelope was correct. I had expected that this modification would significantly increase the proportion of subjects choosing 'five Greens'. I had conjectured that the subjects would use the following reasoning: (i) construct a 'representative event' and then (ii) respond optimally to this event. While the representative event for Problem 1 is "the colors in the five envelopes are two greens, one blue, one yellow and one brown", the representative event in the modified version is "the card in the chosen envelope is Green".

In one graduate economics class a large proportion of students indeed chose (via the Internet) '5 Greens' (61% of the total 49 participants and 33% of the subjects who used the 'matching probabilities' strategy). However, a different class of 46 undergraduates, only 43% chose '5 Greens'. Thus, more research is needed to confirm the conjecture.

4. Extreme diversification in problems with subjective probabilities

When subjects did not receive explicit information about the chances of success of the various alternatives, almost all of them diversified their choices. In other words, they did not follow the rational rule of 'assess the chances of success of each action and choose the most likely one'.

The following problem was presented, via the Internet, to two undergraduate game theory classes in consecutive years (in this case, a very small prize was awarded to a few of the subjects):

Problem 2 ('*Guessing a student's major*')**.** We received a list of all the third-year students (numbering approximately 250) taking a double major in economics and another subject. We selected five of them at random. You have to guess the second major of the five students. You will receive a lottery ticket for each 'hit' (up to a maximum of five tickets). One of the tickets will award the holder a gift coupon of 50 shekels towards the purchase of books.

Beside each guess, a 'window' opened and the subject chose from among the eight most popular second majors and an 'other' option (see ... http://www.princeton.edu/ $\sim$ ariel/99/ failures2_q.html).

In the 1998 class, 28% of the total of 65 students responded with five identical majors and in 1999, only 7% of the 41 students made five identical

selections. Thus, the vast majority of subjects diversified thereby demonstrating irrational behavior.

5. Weak diversification in task-type problems

Once the problem was phrased as the choice of a sequence of five actions, diversification almost completely disappeared.

Problem 3 (‘*Catch the messenger*’)**.** Imagine you are a detective at a shopping center. You know that every day at noon, a messenger arrives with an envelope. The identity of the messenger is unknown; he is one of dozens of messengers who work for a delivery company. The shopping center has four gates and you have only one video camera, which you have to install each morning in one of the four gates. Your aim is to take photos of the maximum number of messengers as they enter the shopping center.

You have to choose a plan determining where to install the camera every morning. You have in hand the results of a reliable statistics on the entry of messengers according to gate: 36% use the Green gate, 25% the Blue gate, 22% the Yellow gate and 17% the Brown gate.

Your plan :
Day : Sun Mon Tue Wed Thu
Gate : __ __ __ __ __

From a class of 50 undergraduates, 70% selected five Greens. When the problem was modified so that the messenger would arrive only once, the percentage of ‘five Greens’ was maintained: 72% (in a group numbering 46).

Comment. In Problem 2, the subjects faced subjective probabilities which encourages diversification. In Problem 3, the problem was a practical one, with objective probabilities and diversification almost disappeared. What happens if the problem is ‘practical’ one and uncertainty is not objectively specified? Undergraduates in economics were asked the following question (in Hebrew):

Problem 4 (‘*Choice of shirt color*’)**.** Imagine that you are working for a company and your manager asks you to purchase an elegant shirt for each of five men who provide services to your company. All five are Tel Aviv-type yuppies in their thirties. The shirts are available in two colors: Burgundy and Black. Their tastes are unknown to you and you must try to choose shirts for the men according to their favorite color.

You estimate that ____% of Tel Aviv’s yuppie population aged 30–40 would choose a Burgundy shirt and __% would choose a Black shirt.

Your choice of colors for the five men is:
(A)__ (B)__ (C)__ (D__ (E)__

Of the 48 students who responded to this question, only 8% chose the same color for all five. Since I asked the subjects to estimate the frequencies of the preference for Burgundy and Black, we were able to observe the alternative 'strategies' used by the subjects more precisely. Eighty percent of the subjects who diversified, did so in proportions approaching the estimated probabilities (± 10%), an outcome which is similar to that of the 'probability matching' strategy.

6. Diversification and randomization

I can think of two reasons for comparing the above results with the behavior of subjects in one-stage decision problems. First, the results can provide a benchmark for comparing the frequencies of diversifiers in the population. Second, it is interesting to compare the diversification phenomenon with its 'twin' harmful randomization in a single stage decision problem.

The following question was presented (via the Internet) to a group of 56 third-year students in a course on game theory:

Problem 5 ('*Meet your friend*')**.** The mall where you wish to meet your friend has four gates. According to a statistical survey, visitors entering the mall choose their gate according to the following proportions:

Gate	North	East	South	West
%	21	27	32	20

You want to meet your friend at the mall but you do not know which gate he is going to use. You can wait for him at only one gate. What are you going to do? (Choose one of the two options and fill in the appropriate details.)

- I will wait at Gate____
- I will choose the gate randomly according to the following probabilities: North: __%; East: __%; South: __%; West: __%

About 69% of the subjects said that they would wait at the South gate. Almost all the others (28%) chose the gates randomly according to the given probabilities. Thus, in spite of the fact that the wording of the question could have suggested to the subjects that they should use a 'mixed strategy', about 70% of the subjects made the 'optimal choice'. I conclude that one cannot expect a higher percentage of 'correct answers' in a multiple-choice problem.

The following similar question was presented to a group of students in 1999:

Problem 5′ (‘*Arrest the suspect*’)**.** You are a police officer wishing to arrest a suspect in a crime. He is about to enter a mall through one of four gates. A statistical survey shows that the proportion of people using each gate is as follows:

Gate	Blue	Green	Red	Yellow
%	21	27	32	20

You have only one policeman to assign to one of the gates according to the outcome of a spin of a roulette wheel. You must assign the proportions of the outcomes to each gate. What will you do?

I will assign the proportion of the outcomes of the spin of the roulette as follows:

Blue: __%; Green: __%; Red: __%; Yellow: __%

In Problem 5′, the pressure on the subjects to choose non-degenerate probabilities was even stronger than in Problem 5 since subjects were forced to divide 100% among the four colors. (Of course, they could still choose a ‘pure action’ by assigning 100% to one option.) In Problem 5′, only 33% of the 43 participants assigned the full 100% to the most likely gate. The rest randomized: 30% chose the matching probabilities and 14% attached equal probabilities to each of the four gates. However, it is quite plausible that the strong tendency to randomize in Problem 5′ was triggered by the fact that the subjects perceived the situation as a zero-sum game (in which the subject wants to escape from the police officer) and not as a decision problem.

7. Related literature

The body of literature most closely related to this paper discusses experiments in which subjects make a sequence of choices without getting any feedback during the process. Gal and Baron (1996) conducted an experiment very similar to Problem 1. They asked subjects to consider the following task: “A die with four red faces and two green faces will be rolled several times. Before each trial the subject will predict which color will show up once the die is rolled”. Subjects were asked to choose a strategy from a list of four possibilities. About 60–75% of the subjects chose the ‘rational strategy’.

In Loomes (1998), subjects were asked to evenly divide 40 balls, 20 Greens and 20 Whites between two bags, A and B. They were told that one of the bags would be randomly selected (bag A with probability 0.65 and bag B with

probability 0.35) and that one of the 20 balls in that bag would be picked. If the chosen ball is green, the subject will receive a prize. Less than 15% of the subjects put all 20 green balls in bag A. The vast majority of subjects put 12–14 green balls in bag A. If one thinks about their choice as the selection of a bag for each of the 20 green balls, then this problem is similar to the modified version of problem 1 in which one of the five envelopes is selected and the subject receives a prize if he chooses that envelope.

Read and Loewenstein (1995) (see also Simonson, 1990) found that when subjects have to select, between products A and B in advance, for a period of several days, they diversify their choices much more than if they had to choose for just one day at a time. The current study diverges from this literature in some fundamental ways. First, in our case, there is always a unique rational answer. This fact rules out the possibility of challenging the results with the simple conjecture that 'individuals diversify because they prefer diversification'. Second, the current study allows us to compare the frequencies of the responses to the rational choice problem with the strategy of 'probability matching'.

There is a large body of literature, which investigates decision makers' strategies in multiple decision problems when the probabilities of 'success' are fixed but unknown to the decision-maker. In this case, subjects get feedback about the outcome of their previous choice before making the next one. This literature actually deals with *learning*. (See, for example, Lee, 1971; Myers, 1976; Vulkan, 2000; Shanks et al., 1999.) The literature identifies two main strategies:

(1) Diversify in the first few stages and eventually 'converge' to always choosing what you perceive as the current option;

(2) Start by 'learning' the probabilities and eventually use the 'probability matching' strategy.

8. Discussion

The main conclusion of this paper is that individuals making a 'multiple decision' very often diversify their choices even though the optimal behavior would clearly require them not to diversify. This tendency is stronger (about 90% of the subjects) in the presence of uncertainty without objective probabilities. The diversification is reduced (to about 60%) when the choice is between 'lotteries' with specified objective probabilities. The proportion of diversifiers is further reduced to about 30% when the subjects have to formulate a plan for a sequence of *actions*. Most of the subjects who diversify follow the 'probability matching' strategy.

As far as I can ascertain, the source of this diversification is a systematic mistake. It is my experience that once I have explained the situation to

subjects, they realize that they have made a mistake. The behavior reported in the paper is, in this respect, similar to non-transitivity, which is considered by subjects to be mistaken once a cyclical choice has been pointed out. However, the fact that a choice reveals a systematic mistake does not make it uninteresting, especially when its appearance is related to characteristics of the problem and when it is committed by subjects who are quite familiar with formal decision problems and probabilistic terms.

Why do we observe a stronger tendency to diversify when uncertainty is not quantified? One possible explanation follows Read and Loewenstein (1995). When uncertainty is vague, people have a strong instinct to 'seek information'. In this case, diversification, although it does not in fact provide information, may be a residual of the instinct to diversify for the sake of learning about the environment in which individuals and groups operate.

Why do we observe a weaker tendency to diversify in problems involving a choice of a string of actions? One possible explanation is that once we consider plans of action, the instinct of preferring a simple course of action begins to work in favor of a constant strategy.

The wide use of the 'probability matching' strategy may be related to the use of the following reasoning: In the face of uncertainty, people construct an event which represents a typical outcome of the random factors involved, following which they optimize given that event. As we know from the literature on the perception of randomness (see Kahaneman and Tversky, 1974), 'People expect that a sequence of events generated by a random process will represent the essential characteristics of that process even when the sequence is short'. People find the typical random sequence of five realizations of uncertainty not to be the one which is the most likely string of events but one which matches the given probabilities (see also Bar-Hillel and Wagenaar, 1991).

As always, an improved understanding of the circumstances in which diversification appears important should only be the first step towards the construction of economic models that explain phenomena which cannot be explained by standard decision theories. Reaching this goal is indeed a challenge.

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