On Price Recognition and Computational Complexity in a Monopolistic Model

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A single seller of an indivisible good operates in a market with many consumers who differ in their ability to process information. The consumers' constraints are modeled in two submodels: the first in terms of the limits on the number of sets in the partition of the price space, and the second in terms of the limits on the complexity of the operation he can use to process a price offer. For the construction of the second submodel, the tool of a "perceptron" is borrowed from the parallel computation literature. Assuming a negative correlation between the seller's cost of supply of the good and the consumer's ability to process information, I demonstrate that the heterogeneity of consumers' abilities can be used by the seller to profitably discriminate among them.

I. Introduction

In almost all models of economic theory, behavioral differences among consumers are attributed to differences in preferences or in the information they possess. In real life, differences in consumer behavior are often attributed to varying intelligence and ability to process information. Agents reading the same morning newspapers with the same stock price lists will interpret the information differently. Even if they do receive the same impressions, the agents may

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differ in their mental ability either to utilize information or to calculate the "optimal" course of action.

In many economic models, asymmetric information regarding market parameters is relevant to decision makers' considerations; economic agents deduce this information from realized equilibrium prices (e.g., the "rational expectations" and the "signaling" models). Usually, the perfect perception of information and the ability to make accurate calculations are assumed. A traditional criticism of these models is that this is a complex operation requiring both skill and comprehensive knowledge of the model. Since the reasoning process is not spelled out in the conventional models, the differing abilities of economic agents in deducing information from prevailing prices do not exist in the conventional analysis. Intuitively, however, such heterogeneity could affect such economic factors as income distribution and help explain the rationale of economic institutions whose existence is dependent on these differences.

This paper is devoted to the construction of a simple economic model in which decision makers differ in their ability to process the information given in a price offer made in the market. The reader may wonder why there would be any difficulty in recognizing and processing a posted price; after all, a price is only a number. However, recall that it is rare that an offer is indeed given as just one number. Usually, an offer is composed of a long list of elements corresponding to features such as the exact characteristics of the product, the payment arrangements, and the warranties. The multiplicity of such details makes the calculation of "the price number" a nontrivial task. Furthermore, if the price depends on a state of nature, the decision maker may be interested in making complicated inferences from the price about the prevailing state of nature.

How could one model differences in abilities to process information? The approach taken in this paper is that while differences in information may be modeled by differences in partitions of the relevant state space, differences in the ability to process information may be modeled by the differences in the constraints on the family of partitions available to the individuals. Since a decision may depend only on the cell in the partition, this limits the set of response functions available to the consumers.

Recall that a *rational* economic person is a creature who is not restricted in his or her ability to process information or to make calculations. Embedding such restrictions in economic models is impossible unless we enrich the model with details on the reasoning procedures used by economic agents in their decision-making processes. The expansion of the established body of economic analysis to encompass the procedural aspects of decision making (Simon 1982) is the hallmark of so-called *bounded rationality;* as such, the current paper can be viewed as a move in this direction.

The specific economic model that is the cornerstone of the paper is described in Section II. It is a model of a monopolist who sells an indivisible good (services) so that the cost of producing a unit of the good depends on the consumer. Sections III and IV include two extensions of the model in which two types of restrictions on the ability of the consumers to process information are introduced.

The economic model of Section II and the "bounded rationality" elements introduced in Sections III and IV are admittedly arbitrary and should be taken only as examples. In line with what I consider the objective of economic theory, the paper is aimed mainly at the exposition of a structure of equilibrium with heterogeneity of reasoning processes. No claim is made beyond the clarification of the logic of the equilibrium under these circumstances. Let us now turn to a detailed description of the basic model.

II. The Basic Model

Consider a market for a single good consisting of a single producer and N consumers, each of whom is interested in consuming only one unit of the commodity. The economic parameters of the market depend on a state of nature that may be either H or L. All agents share the initial belief that the probabilities of the states H and L are π_H and π_L, respectively. The information on the realized state of nature is delivered exclusively to the seller. In state L, the seller's production cost, c_L, is zero, regardless of the quantity and the identity of the buyers. In state H, the seller's marginal cost depends on the type of the consumers who purchase the commodity. The consumers are divided into two types: N_1 consumers are of type I, for which the cost of production is c_1, and N_2 consumers are of type II, for which the cost of production is c_2 ($N = N_1 + N_2$). A consumer purchases the good if and only if the expected surplus value is strictly positive. The surplus derived from consuming one unit of the commodity for the price p is $v_L - p$ if the state of nature is L and $v_H - p$ if the state of nature is H. It is assumed that $c_1 > v_H > c_2 > v_L > 0$ (see fig. 1). The assumption is that a strictly positive expected surplus is required in order to rule out equilibria in which consumers will differentiate their behavior arbitrarily, that is, without having economic reasons to behave differently.

Events occur in the market in the following order: (1) The seller announces a price policy that is a specification of a "lottery" of prices (a probability measure with finite support) for each of the states of nature. The seller's announcement is a *commitment* to supply whatever

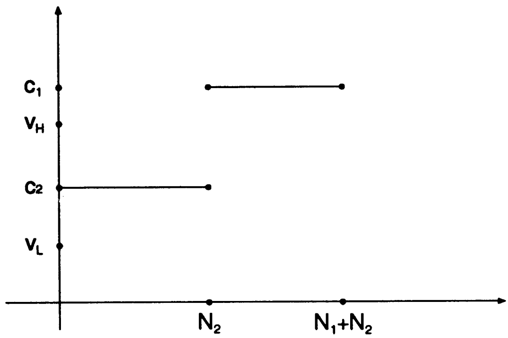

FIG. 1

quantity of the good is demanded by the consumers at the price resulting from the lottery following emergence of the state of nature. Thus the announcement of the seller forces all fully rational consumers to hold the same beliefs on the state of nature after the realization of the price. (2) Nature selects the state of nature, and the seller's offer is determined by the probabilistic device to which the seller has committed himself. (3) The consumers are informed about the realization of the lottery. On the basis of the posted price (in later sections, the consumer information about the posted price will be restricted) and the announced pricing policy, each consumer has to make a decision whether to accept or to reject the offer.

To summarize, the model is a conventional Stackelberg, leader-follower situation in which the seller is the leader who chooses the pricing policy and the consumers are the followers who choose acceptance rules. Later, the choice of the acceptance rules will be restricted to reflect computational complexity constraints, and the two types of consumers will differ in the sets of acceptance rules that they are able to use.

Remarks.—(1) The seller's strategy is the choice of a random device for every state of nature. Although the seller employs random devices, his strategy is a *pure* strategy, not a mixed strategy. The strategy (including the random devices that are part of it) determines the consumers' behavior; in equilibrium, the seller may strictly prefer a strategy with stochastic elements over a strategy that specifies a deterministic price for each state of nature. Recall that in a mixed-strategy equilibrium, in contrast, a player has to be indifferent to all deterministic strategies that lie in the support of his mixed strategy. (2) Notice that given the consumers' purchasing strategies, the seller may be better off by not following the announced pricing policy. However, in the current model, the seller is committed to the policy he has announced, and the posted price *must* be determined according to the outcome of the random device that the announced strategy assigns to the realized state of nature.

The realization of the posted price not only determines the terms of trade but also reveals information about the state of nature if the lotteries that correspond to the different states of nature are not identical. The seller's basic dilemma is that at state H he cannot gain from selling the good to the type I consumers since his cost of producing the good for those consumers is higher than their reservation value. It is assumed further that conditional on the state H, the seller prefers not to sell more than N units, even for the maximal price of v_H, that is, $Nv_H < N_2c_2 + N_1c_1$.

Ideally, at state H, the seller would sell the good only to the type II consumers. However, the seller, the exclusive information holder, cannot distribute the information among only some of the participants in the market. Distributing information about the real price can be done, in this model, only via the price mechanism; without additional heterogeneity the price mechanism does not enable the seller to discriminate between agents, and the seller's bound on his expected profits is $\Pi* = \pi_L N v_L$. To see that he can (almost) achieve this level of profits, notice that when the seller charges $v_L - \epsilon$ (with probability one) in state L and charges a very high price in state H, his expected profits are arbitrarily close to $\pi_L N v_L$. Let us verify that the seller cannot achieve higher profits by any other price strategy (including those that employ random devices). For any price p that accords with the seller's strategy and that is accepted by the buyers, $p < \text{prob}(H|p)v_H + \text{prob}(L|p)v_L$, the revenues cannot exceed $\text{prob}(H|p)Nv_H + \text{prob}(L|p)Nv_L$ and the expected production costs are

$$\text{prob}(L|p)Nc_L + \text{prob}(H|p)N_2c_2 + \text{prob}(H|p)N_1c_1.$$

Thus the seller's profits are bounded by

$$\text{prob}(L|p)Nv_L + \text{prob}(H|p)[N_2(v_H - c_2) + N_1(v_H - c_1)].$$

According to our assumptions, $N_2(v_H - c_2) + N_1(v_H - c_1) < 0$; thus every price in equilibrium that is accepted by the buyers at state H contributes to the seller's profits less than $\text{prob}(L|p)Nv_L$. Integrating over all p that are offered by the seller's strategy and are accepted by the consumers, we see that the seller's total profits are bounded by $\pi_L N v_L$.

Needless to say, the outcome of the seller's strategy is inefficient. In state H, the seller underproduces (or does not produce at all), even though it is mutually beneficial for the seller and type II consumers that the seller produces and sells the commodity to these consumers for any price below v_H and above c_2.

III. Imperfect Price Recognition

We are ready to add a new feature to the model: the imperfection in the consumers' calculations. Assume that type I consumers are able to determine only *one* cutting point; that is, they can split the price space into only two *connected* sets and are able to attach the order either "buy" or "don't buy" to each of the two sets. (If the requirement that the sets are connected is omitted, the rest of the restriction will become powerless. The fact that the consumers have to choose between two actions makes a [not necessarily connected] two-set partition sufficient for implementing the best response.) In other words, type I consumers are able to make decisions of the following types: "buy iff $p \leq p^*$," "buy iff $p < p^*$," "buy iff $p \geq p^*$," "buy iff $p > p^*$," "always buy," and "never buy." Type II consumers, on the other hand, are able to determine *two* cutting points that split the price space into up to three connected sets. This means that a type II consumer can also adopt an acceptance rule of the type "buy (or don't buy) the commodity if the price lies in a certain interval and don't buy (or buy) the commodity if the price lies outside the interval."

The selection of the partition and the action conditional on the received information is carried out by each of the consumers between stages 1 and 2, that is, after the buyers learn the announced pricing policy and before the realization of the price. The decision concerning the partition is subject to the restrictions imposed by the consumer's type. To summarize, events in the model occur in the following order: Stage 1: The seller announces a pricing policy. Stage 2: Each consumer selects a partition (given the constraints determined by the consumer's type). Stage 3: Nature selects the state and the price is determined. Stage 4: Each consumer gets information about the cell in his partition, which includes the announced price, and decides whether or not to purchase the good.

Notice that the assumption that the cost of production for a type I consumer is higher than that of a type II consumer narrows down the set of situations covered by the example. This correlation between the production cost and a customer's ability to process information fits especially commodities such as education and advisory services.

It will be shown that the seller can utilize the differences between type I and type II consumers to derive profits arbitrarily close to $\Pi^* = \pi_L N V_L + \pi_H N_2(V_H - c_2)$. The idea is quite simple. Choose ϵ_L and ϵ_H so that $\pi_L \epsilon_L > \pi_H \epsilon_H$ and consider the following pricing strategy: in state H charge the price $v_H - \epsilon_H$ with probability one; in state L

charge the price $(v_H + v_L)/2$ with low probability and $v_L - \epsilon_L$ with high probability. Given this strategy, a type II consumer is able to partition the price space $\{v_L - \epsilon_L, (v_H + v_L)/2, v_H - \epsilon_H\}$ into three sets and to purchase the good only at the high and low prices. A type I consumer is deterred by the loss incurred if he buys the commodity for the price $(v_H + v_L)/2$ in state L. He can purchase the good for a price either not higher than $v_L - \epsilon_L$ or not lower than $v_H - \epsilon_H$. Since $\pi_L\epsilon_L > \pi_H\epsilon_H$, the former is better for the consumer; thus by choosing small enough ϵ_L and ϵ_H, the seller can approach Π^*, the maximal profit.

IV. Parallel Computation

In the previous section, a seller's offer was a price. In this section, a seller is allowed to split a price into several components; an offer is a K-tuple $(p_1, \ldots, p_K)$, where the number p_k is the price of the kth component of the commodity. The meaning of the acceptance of an offer of the vector $\mathbf{p}$ is that the consumer gets one unit of the commodity in exchange for Σp_k units of money. Splitting a price into several parts is quite common in real markets: for example, when we buy a stereo set, we usually get a list of the items' prices as well as the amount of tax and various service fees. A consumer who accepts the seller's offer p pays Σp_k; however, the manner in which the sum Σp_k is divided into the K components may contain relevant information concerning market conditions. Agents may experience difficulty in decoding the information and may differ in their ability to interpret the information contained in the offer.

A consumer's strategy will have to process a vector of numbers that compose an offer. The consumer's strategy will be modeled by a computational device called a "perceptron." (For an outstanding introduction to this notion, see Minsky and Papert [1988].) A perceptron is a collection of processors that operate in parallel on the realized price vector and send a number to the center. The center sums up the numbers and makes a decision on the basis of whether the sum is above or below some fixed threshold number.

Formally, define a processor to be a real function ϕ that receives some of the components of the price vector as its input and gives a real number as an output. A perceptron is a collection of processors $\phi_1, \ldots, \phi_M$ and a threshold number α^*. The consumer purchases the commodity if and only if $\Sigma\phi_m \leq \alpha^*$. A consumer's purchasing strategy is the choice of the perceptron. Figure 2 shows a schematic illustration of the computational device ascribed to the consumers.

We are now ready to add the imperfection in the consumers' calculations to the model. The consumers are bounded by the complexity

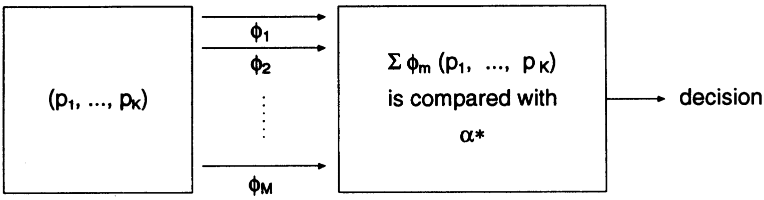

FIG. 2

of the processors that they are allowed to use. The complexity of a perceptron is measured by its order, that is, the number of price vector components in its domain. For example, if ϕ depends on only one of the p_k's, then ϕ is a processor of order 1; if it is a function of two prices, the processor is of order 2. The consumers have no restriction on the size of M and have a perfect ability to compare the outcome of the sum of M numbers with the threshold level α^*. When the calculated sum of the processors' values is below the threshold level, the consumer accepts the offer; when it is above it, the consumer rejects the offer.

I wish to emphasize once again that, obviously, there is no claim that this computational device and complexity measure are in any sense a part of the "true" description of the human processing of a vector of price components. The formal tool of perceptrons is taken as a vehicle for building an additional example of the constraints on the information process. Nevertheless, the concept is not meaningless and does reflect basic intuitions. It fits, for example, a case in which a decision maker considers a change in the status quo and uses a procedure in which (1) he asks a group of experts (or interested parties) to send reports to the center on the amount of support for the change, (2) he sums up the support noted in the reports, and (3) he takes action only if the support exceeds a certain threshold.

The complexity of the process is measured here by the complexity of a single processor. The difficulty of the basic operation creates the bound on the consumer's strategy. An analogue to that approach would be an assumption that only the type of processor used in a personal computer system, and not the number of computers in the network, is the measure of the complexity of the network.

It is assumed that consumers differ with respect to the order of perceptrons that they are able to employ. Type II consumers are able to employ order 2 perceptrons, and type I consumers are constrained to use only order 1 perceptrons.

As in the previous section, the functional difference between the two types of consumers depends on the variety of prices existing in the market. Obviously, if there are at most two prices in the market, the two types will be able to function equally well. In contrast, if all prices are possible, a type I consumer is able to execute a policy of purchasing the commodity if and only if the total price is precisely some value p^*. The proof is quite simple and may be found in Minsky and Papert (1988). Nevertheless, a type II consumer is able to pursue such a strategy since $\Sigma p_k = p^*$ is equivalent to

$$(\Sigma p_k - p^*)^2 = \sum_{k,l} p_k p_l - \sum_{k} 2p^* p_k \leq 0$$

and all $p_k p_l$ and $-2p^* p_k$ are order 1 or 2 perceptrons.

Let us summarize the structure of the model. The seller first announces a pricing policy that assigns a lottery of price vectors to every state. As before, the seller is committed to that policy. Next, every consumer has to choose his purchasing strategy (constrained by his type). Finally, the price vector is realized and the consumers implement their purchasing policies.

We shall now see that by utilizing the differences between the two types, the seller can achieve the same level of expected profits, Π^*. Consider the following pricing strategy.

The seller splits the price of the commodity into $K = 2$ parts. In state H the seller chooses the vector $(p, p) = ((v_H/2) - \epsilon_H, (v_H/2) - \epsilon_H)$ with probability one. In state L he chooses the vector $(q, q) = ((v_L/2) - \epsilon_L, (v_L/2) - \epsilon_L)$ with probability $1 - \delta$ and each of the vectors $(p, q) = ((v_H/2) - \epsilon_H, (v_L/2) - \epsilon_L)$ and $(q, p) = ((v_L/2) - \epsilon_L, (v_H/2) - \epsilon_H)$ with probability $\delta/2$. A type II buyer is able to escape the trap of purchasing the good for the price $p + q$ at state L by having an order 2 perceptron that gives the value minus one for the vectors (p, p) and (q, q) and the value one for the states (p, q) and (q, p), setting $\alpha^* = 0$. (Alternatively, he can choose the strategy "accept (p_1, p_2) iff $p_1^2 + p_2^2 - 2p_1p_2 = [p_1 - p_2]^2 \leq 0$," which requires perceptrons of order 1 or 2 only.) A type I buyer cannot pursue a strategy in which he buys the commodity only at the price vectors (p, p) and (q, q). If such a purchasing strategy existed, there would be *two* perceptrons (because of the arbitrariness of the perceptrons, more than two will not help), ϕ_1 and ϕ_2, and a number α^* so that

$$\phi_1(q) + \phi_2(q) \leq \alpha^*,$$

$$\phi_1(p) + \phi_2(p) \leq \alpha^*,$$

$$\phi_1(p) + \phi_2(q) > \alpha^*,$$

$$\phi_1(q) + \phi_2(p) > \alpha^*.$$

These four inequalities clearly result in a contradiction.

Now for any number δ we can choose an ϵ_H and ϵ_L so small that

$(1 - \delta)\pi_L \epsilon_L > \pi_H \epsilon_H$, which guarantees that the consumer will prefer to avoid the possibility of purchasing the commodity with a probability of $\pi_L \delta/2$ for the price $p + q$ even if he buys the commodity at the state H for the price $p + p$, and will prefer to purchase the good for the price $q + q$ in state L rather than purchase the good for the price $p + p$ in state H.

Notice that the seller's strategy uses four price vectors: (p, p), (q, q), (p, q), and (q, p). A type II consumer can utilize the partition of the set of four price vectors $\{\{(p, p), (q, q)\}, \{(p, q), (q, p)\}\}$, but a type I consumer cannot. A type I consumer can use a partition such as $\{\{(p, p), (p, q), (q, p)\}, \{(q, q)\}\}$ by selecting the rule of buying the commodity if the sum of the components is not more than $p + q$ (this is done by utilizing the two perceptrons $\phi_i(p) = p_i$ and setting $\alpha^* = p + q + \epsilon$). Similarly, a type I consumer can utilize the partition $\{\{(q, q), (q, p), (p, q)\}, \{(p, p)\}\}$. He can also utilize the partition $\{\{(p, q)\}, \{(p, p), (q, p), (q, q)\}\}$ by choosing $\phi_1(p) = 2$, $\phi_2(q) = 2$, $\phi_1(q) = -3$, $\phi_2(p) = -3$, and $\alpha^* = 0$. But, as was shown above, such a consumer is not able to utilize the partition $\{\{(p, p), (q, q)\}, \{(p, q), (q, p)\}\}$, which is the only partition that would enable him to increase his payoff above what he is achieving by utilizing perceptrons of order 1.

V. Related Literature

The endogenous choice of the partition of the set of possible prices has been previously modeled by Dow (1991). His model is a single decision-making problem that is not embedded in an equilibrium analysis. Dow analyzes a two-stage "search" model in which a decision maker receives information about the prevailing price of a certain good in two stores in a predetermined sequential order. The decision maker cannot remember the exact price that he observed in the first store when he arrives at the second store. His aim is to partition the potential price space so that the partition will provide him, "on average," with the most useful information for reaching the decision from which store to buy the good, a decision that, by assumption, he must make after observing the second price. Dow presents necessary conditions for the optimal partition.

Also relevant is the literature on equilibrium in markets with search. In these models, a consumer makes his purchasing decision through a process of search. The structure of equilibrium in such models reflects the heterogeneity in consumers' search costs. The search process is not necessarily a physical search but can be thought of as a model of a mental process in the consumer's mind. Consequently, the search costs can be interpreted as the costs associated with the searcher's difficulties in recognizing prices, as opposed to physical sampling costs. Within the literature attempting to explain "price dispersion," the closest models are those of Salop (1977) and Salop and Stiglitz (1977). In these models, all consumers know the prices available in the market but they do not know what store charges what price. A consumer has to choose either to purchase the good at random or to spend an exogenously given cost in obtaining the information about the location of the store charging the lowest price. There is heterogeneity among the consumers regarding the cost associated with getting that information. Assuming a correlation between the consumer's search cost and other consumer characteristics, Salop shows that the model presented allows for an optimal strategy for a monopolist where more than one price is charged. In Salop and Stiglitz, there are many sellers; the consumers bear a "search cost" for acquiring the information about which stores are charging which prices. The possibility of an equilibrium with price dispersion is then demonstrated.

VI. Conclusion

This paper has presented a simple model in which the heterogeneity of consumers with respect to their ability to process information correlated with economic factors is utilized by a monopolist to derive additional profits. In the two versions of the model, the monopolist forces type I consumers to focus attention on escaping the trap that he has prepared for them by offering a (sometimes) high price in the state of nature L. Being occupied with this task, a type I consumer cannot devote his computational resources or attention to the task of identifying the conditions under which it is desirable for him to purchase the commodity for a high price. In contrast, a type II consumer is able to infer the true state of nature from the monopolist's pricing strategy and is able to both escape the trap and identify the conditions under which paying a high price is profitable.

As in many other "bounded rationality" models, we have limited the ability of consumers on one aspect but, at the same time, we have required more "sophistication" from them on another: although agents are assumed to be bounded in their ability to perceive prices, they are not assumed to be constrained in their ability to arrive at the optimal strategy required to choose the partition used in perceiving prices.

Within the context of industrial organization, this paper shows that the complexity of the price scheme can be used strategically by price setters. A casual observation of real life confirms that price schedules (or the characteristics associated with products) are very complex and that the complexity of the price structure affects the group of economic agents who are active in a given market (e.g., consumers trading in financial markets).

Whatever the case, the principal aim of this paper is more abstract. In contrast to other models in which agents possess different information about the state of nature, here the agents differ in their ability to absorb information on the endogenous *equilibrium prices.* It is challenging to study richer equilibrium models in which the agents' behavior depends on their ability to process the information embedded in equilibrium prices. Such models may constitute a response to the criticism concerning the assignment of complicated computational tasks to economic agents.

Finally, the model is a simple example of an economic model with elements of bounded rationality. In spite of the arbitrariness, I hope that the paper suggests some useful modeling ideas.

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