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Similarity and Decision-making under Risk (Is There a Utility Theory Resolution to the Allais Paradox?)

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It is argued that the Allais paradox reveals a certain property of the decision scheme we use to determine the preference of one lottery over another. The decision scheme is based on the use of similarity relations on the probability and prize spaces.

It is proved that for every pair of similarity relations there is essentially only one preference consistent with the decision scheme and the similarities. It is claimed that the result shows a basic difficulty in reconciling utility theory with experimental data. *Journal of Economic Literature* Classification Number: 026. © 1988 Academic Press, Inc.

1. INTRODUCTION

The experimental work on choice under risk provides evidence that human behavior is very often inconsistent with expected utility theory. The results seem to be strongly supported by our own "thought experiments." The strength of the results is demonstrated by the fact that these phenomena are often deemed paradoxical.

Critiques of expected utility theory as a descriptive theory have led countless scholars to alter the von Neumann–Morgenstern axioms in order to establish alternative utility forms which would be consistent with the experimental evidence.

In the traditional approach a list of axioms is suggested and is proven to imply a certain functional form of the utility representations. Some of these axioms are consistency requirements in the sense that they require dependencies of the preference over different pairs of elements. A common axiom is the independence axiom: if $L_1 \gtrsim L_2$ then for all t and for all L,

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$t \cdot L_1 \oplus (1-t) \cdot L \gtrsim t \cdot L_2 \oplus (1-t) \cdot L$ (the symbol $\oplus$ denotes the probability mixture operation). The intuitive argumentation for such an axiom is as follows: *if* a subject is asked about the validity of the independence axiom, would he not agree that he does satisfy the axiom? The reasonableness of the axiom depends on the answer to a hypothetical question which ordinary people (including experts) do not ask and are not aware of.

The approach of this paper is different in the sense that the main axiom is derived from what seems to be a part of the decision-making procedure. Deriving the axioms from the natural process used by people rather than using artificial axioms is a more promising strategy for constructing a descriptive theory.

For concreteness let us consider a decision maker who has to choose one lottery from each of several pairs of lotteries. Assume all lotteries are of a special simple kind (x, p) with the prize x (dollars) having probability p and the prize 0 (dollars) probability $1-p$.

The procedure for determining the preference studied here is exposed by the following experiment which is a variant of the Allais ratio paradox (see Allais [1]).

Let

$$L_1 = (4000, 0.8), \qquad L_2 = (3000, 1)$$

$$L_3 = (4000, 0.2), \qquad L_4 = (3000, 0.25).$$

When asked to choose between pairs of lotteries (see Kahneman and Tversky [4]) the vast majority of subjects answered that $L_1 < L_2$ and $L_3 > L_4$, violating the independence axiom ($L_3 = 0.25 \cdot L_1 \oplus 0.75 \cdot 0$ and $L_4 = 0.25 \cdot L_2 \oplus 0.75 \cdot 0$).

A possible explanation for these preferences is the following: the preference of L_2 over L_1 is due to risk aversion. When comparing L_3 and L_4 the probabilities 0.2 and 0.25 are evaluated to be similar (in contrast to 0.8 and 1 which are not similar), the prizes \$4000 and \$3000 are not considered by the subjects to be similar, and the size of the prizes becomes the decisive factor.

Thus to my understanding, the Allais paradox reveals a certain property of the decision scheme used by people in the determination of preferences between pairs of lotteries. The decision scheme is based on the use of similarity relations on the probability and prize spaces. Given two lotteries (x_1, p_1) and (x_2, p_2) the requirement of the decision procedure is that it is initiated by checking the validity of the two statements "x_1 is similar to x_2" and "p_1 is similar to p_2." If only one of the two statements is true in the decision maker's perception, for instance x_1 is similar to x_2, then the probability dimension becomes the decisive factor. If in addition $p_1 > p_2$

then (x_1, p_1) is determined as preferable to (x_2, p_2). If neither or both of the statements are viewed to be correct then the decision process is not restricted.

The above procedure explains some other famous paradoxical examples like the classical Allais paradox. In this example a decision maker is asked to choose between a sure chance of \$1,000,000 and a 10:89:1 chance of \$5,000,000:\$1,000,000:\$0 and then between a 10:90 chance of \$5,000,000:\$0 and a 11:89 chance of \$1,000,000:\$0. The first common preference of the sure lottery is explained by risk aversion while the latter preference of the 10:90 chance lottery is explained by the use of the above procedure.

In this paper we will derive the consequences of the decision scheme on the set of preferences that are consistent with it. The main finding is that *similarities* on both the prize and the probability dimensions result in a "unique" preference which is consistent with the similarities and the above procedure. Thus there is little room for other considerations to affect the determination of a preference. An examination of the determination of the preference L_2 over L_1 in the above example reveals that when neither x_1 is similar to x_2, nor p_1 similar to p_2, we use other criteria to determine the preference. I find it hard to believe that such other considerations will coincide with the "unique" preference relation consistent with the similarities. Therefore the above result shows a basic difficulty in reconciling utility theory with decision-making procedures as described above.

The paper is in line with works of Amos Tversky in two aspects. First, it emphasizes the role of similarities in human reasoning (see Tversky [9]). Second, it points out that actual decision procedures may lead to non-transitivity (see Tversky [8]). The work is also related to previous works by Luce ([5, 6]) and Ng [7].

2. The Basic Concepts

2.1. *Preferences on the Set of Lotteries*

Denote by (x, p) a lottery which gives prize x with probability p and the prize 0 with probability $1-p$, where $0 \leqslant x \leqslant 1$. The set $X \times P = [0, 1] \times [0, 1]$ is the set of lotteries. Denote by $\gtrsim$ a binary relation on the set of lotteries. $L_1 > L_2$ means that lottery L_1 is strictly preferred to lottery L_2. It is assumed that $\gtrsim$ satisfies the following assumptions:

(R-1) The relation is transitive and reflexive.

(R-2) The relation is monotonic: $x_1 > x_2$ and $p_1 > p_2$ imply that $(x_1, p_1) > (x_2, p_2)$.

(R-3) The relation is continuous.

So far the set $X \times P$ has no meaning beyond being a square. Under the above interpretation of (x, p) as a lottery there is no difference between any of the pairs $(x, 0)$ and $(0, p)$. Therefore we assume:

(R-4) For all x and p, $(x, 0)$ and $(0, p)$ are indifferent to $(0, 0)$.

Utility theory states that there is a function $U: X \times P \to R$ satisfying $L_1 > L_2$ iff $U(L_1) > U(L_2)$. The expected utility theory hypothesis is that there exists a function $u: X \to R$ such that $U(x, p) = pu(x) + (1 - p)\, u(0)$.

2.2. *Similarity*

A binary relation $\sim$ on the set $A = [0, 1]$ is a *similarity* relation if:

(S-1) For all $a \in A$, $a \sim a$.

(S-2) For all $a, b \in A$, if $a \sim b$ then $b \sim a$.

(S-3) Continuity: the graph of $\sim$ is closed.

(S-4) Betweenness: if $a \leqslant b \leqslant c \leqslant d$ and $a \sim d$ then $b \sim c$.

(S-5) Non-degeneracy:

(1) For all $0 < a < 1$ there are b and c, $c < a < b$, such that $b \sim a$ and $c \sim a$. For $a = 1$ there is c as above. Thus the only element which may not be similar to any other element in A is zero.

(2) $0 \not\sim 1$.

Notation. $a^* = \max\{b: b \sim a\}$ and $a_* = \min\{b: b \sim a\}$. Notice that unless $a^* = 1$, $(a^*)_* = a$, and unless $a_* = 0$, $(a_*)^* = a$.

(S-6) Responsiveness: a^* and a_* are strictly increasing functions at any point where they get a value in the open interval $(0, 1)$.

EXAMPLES. The ε-difference similarity is defined by $x \sim y$ if $|x - y| \leqslant \varepsilon$. The λ-ratio similarity is defined by $x \sim y$ if $1/\lambda \leqslant x/y \leqslant \lambda$.

The concept of similarity as defined here is closely related to Luce's concept of semi-order (see Luce [5]). If $\sim$ is a similarity relation then the binary relation P defined by aPb if $a > b$ and $a \not\sim b$ is a semi-order.

Remark. Avishai Margalit pointed out to me that in the natural language a similarity relation does not satisfy the symmetry requirements. Therefore, in the context of this paper it could be better to use the phrase "*a* is approximately the same as *b*" rather than "*a* is similar to *b*."

2.3. *Presentation of Similarity Relations*

Let $H(a)$ be a strictly increasing and positive function on the unit interval and let λ be a number strictly greater than 1. The relation $\sim$ defined by

$$a \sim b \quad \text{if} \quad 1/\lambda \leqslant H(a)/H(b) \leqslant \lambda$$

is a similarity relation. It is said that the pair (H, λ) represents the similarity relation $\sim$.

The next proposition states that a similarity relation has a representation. The proposition is close to previous representation theorems of semi-orders. For an excellent survey of those theorems see Fishburn [3].

PROPOSITION 1. *For all similarity* $\sim$ *(satisfying* (S-1) *to* (S-6)) *and for all* $\lambda > 1$ *there is a function* H *such that* $a \sim b$ *iff*

$$\frac{1}{\lambda} \leqslant \frac{H(a)}{H(b)} \leqslant \lambda.$$

Proof. Define inductively a sequence (x^n) such that $x^0 = 1$ and $x^{n+1} = (x^n)_*$. By (S-3), x^{n+1} is well defined. By (S-5), $x^n \to 0$. Define $H(1) = 1$. Define H on the interval $[x^1, x^0]$ as any strictly increasing function satisfying $H(x^1) = 1/\lambda$ and $H(x^0) = 1$. For $x \in [x^{n+1}, x^n]$ define $H(x) = H(x^*)/\lambda$. By (S-6) the function H is strictly increasing. If $0 \not\sim x$ for all $x \neq 0$ define $H(0) = 0$. (Otherwise there is an n such that $x^n = 0$.)

We still have to verify that (H, λ) represents $\sim$. First assume $1/\lambda \leqslant H(a)/H(b) \leqslant \lambda$ and $a \leqslant b$. Then by the construction of H, $b \leqslant a^*$. By (S-4), $a \sim b$. Second assume $a \sim b$. By (S-4), $a_* \leqslant b \leqslant a^*$. By the construction of H, $H(a) = H(a^*)/\lambda$ unless $a^* = 1$, where $H(a) \geqslant H(a^*)/\lambda$. Unless $a_* = 0$, $a = (a_*)^*$ and $H(a) = H(a_*)\lambda$. If $a_* = 0$, $H(a) \leqslant H(a_*)\lambda$. Thus, $H(a)\lambda \geqslant H(a^*) \geqslant H(b)$ and $H(b)/H(a) \leqslant H(a^*)/H(a) \leqslant \lambda$. Similarly, $H(b) \geqslant H(a_*) \geqslant H(a)/\lambda$. Therefore, $1/\lambda \leqslant H(b)/H(a) \leqslant \lambda$.

3. A PROCEDURE FOR PREFERENCE DETERMINATION

In this section we will analyze a procedure for preference determination. We will refer to such a procedure as $*$ procedure. Let $L_1 = (x_1, p_1)$ and $L_2 = (x_2, p_2)$ be two lotteries. Coming to determine the preferred lottery, an individual is assumed to go through the following steps:

step 1: If both $x_1 > x_2$ and $p_1 > p_2$ then $L_1 > L_2$.
If this step is not decisive move to step 2.

step 2: If $p_1 \sim p_2$, $x_1 \not\sim_x x_2$, and $x_1 > x_2$ then $L_1 > L_2$.
If $p_1 \not\sim_p p_2$, $x_1 \sim_x x_2$, and $p_1 > p_2$ then $L_1 > L_2$.
If this step is not decisive then move to step 3 which is not specified.

DEFINITION. A preference $\gtrsim$ is said to satisfy codition $*$ relative to the similarities $\sim_x$ and $\sim_p$ if for all x_i, $p_i > 0$:

(1) Whenever $p_1 \sim_p p_2$, $x_1 \nsim_x x_2$, and $x_1 > x_2$ then $(x_1, p_1) > (x_2, p_2)$.

(2) Whenever $p_1 \nsim_p p_2$, $x_1 \sim_x x_2$, and $p_1 > p_2$ then $(x_1, p_1) > (x_2, p_2)$.

Let C be the set of all $(\sim_x, \sim_p, \gtrsim)$ such that $\gtrsim$ satisfies both (R-1)–(R-4) and $*$ relative to similarities $\sim_x$ and $\sim_p$ satisfying (S-1)–(S-6).

EXAMPLE. Let $\gtrsim$ be a preference represented by the utility function $x^{\alpha}p$. Let $\sim_x$ and $\sim_p$ be the λ and the λ^{α} ratio similarities. Then $\gtrsim$ satisfies $*$ relative to $\sim_x$ and $\sim_p$. (If $p_1 \sim_p p_2$, $x_1 \nsim_x x_2$, and $x_1 > x_2$ then $x_1^{\alpha} p_1 > (x_2 \lambda)^{\alpha} p_1 = x_2^{\alpha} \lambda^{\alpha} p_1 \geqslant x_2^{\alpha} p_2$.)

It should be mentioned that although they are different there are common factors between the above procedure and procedures suggested in Encarnacion [2] and in Luce [6].

LEMMA 1. *If $(\sim_x, \sim_p, \gtrsim) \in C$ then there is no strictly positive x or p such that $0 \sim_x x$ or $0 \sim_p p$.*

Proof. Assume $x > 0$ and $x \sim_x 0$. By (S-5) on $\sim_p$, $1_* \neq 0$. By $*$ for all $\varepsilon > 0$, $(x, 1_* - \varepsilon) < (0, 1)$, and by (R-3), $(x, 1_*) \lesssim (0, 1)$. By (R-4), $(0, 1) \sim (0, 0)$. By (R-1), $(x, 1_*) \lesssim (0, 0)$, contradicting the monotonicity (R-2).

Remark. By Lemma 1 there is no preference relation which satisfies $*$ relative to difference similarities. Obviously this is not correct unless (R-4) is assumed.

The rest of the paper is devoted to a study of the set C. It will be argued that $*$ puts a very tight restriction on the set of preferences consistent with a pair of similarities. The main conclusion of the paper is that if an individual uses a $*$ procedure and if his choice is transitive then independently of what he is doing in step 3, the preference relation he produces is "almost" unique and represented by a utility function $g(p)\,u(x)$. Thus the information about the individual's similarity relations and the information that he uses a $*$ procedure "almost" characterize a decision maker and leave very little room for any other considerations.

PROPOSITION 2. *Let $\sim_x$ and $\sim_p$ be similarities satisfying that there is no $x > 0$ or $p > 0$ such that $0 \sim_x x$ or $0 \sim_p p$. Then there are functions $u\colon X \to R$ and $g\colon P \to R$ such that:*

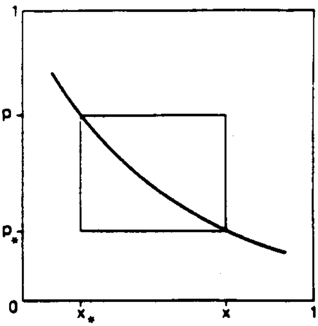
(a) *The function* $g(p)\,u(x)$ *represents a preference on* $X \times P$ *satisfying* (R-1)–(R-4), *and satisfies* $*$ *relative to* $\sim_x$ *and* $\sim_p$.

(b) *If* $(\sim_x, \sim_p, \gtrsim) \in C$ *then for all* (x_1, p_1) *and* (x_2, p_2) *satisfying* $g(p_1)\,u(x_1) > g(p_2)\,u(x_2)$ *there are* $x_i' \sim x_i$ *and* $p_i' \sim_p p_i$ *such that* $(x_1', p_1') > (x_2', p_2')$.

Proof of (a). By Proposition 1 and Lemma 1 there are continuous and strictly increasing functions u and g satisfying $u(0) = g(0) = 0$ and a number $\lambda > 1$ which represents $\sim_x$ and $\sim_p$ accordingly. Let $\gtrsim$ be a preference represented by $g(p)\,u(x)$. It will be proved that $(\sim_x, \sim_p, \gtrsim) \in C$. Assume $x_1 \sim_x x_2$, $p_1 \not\sim_p p_2$, and $p_1 > p_2$. Then $1/\lambda \leqslant u(x_1)/u(x_2)$ and $\lambda < g(p_1)/g(p_2)$. Therefore $g(p_1)\,u(x_1) > g(p_2)\,u(x_2)$.

LEMMA 2. *Let* $(\sim_x, \sim_p, \gtrsim) \in C$. *For all* (x, p) *such that* $x_* > 0$ *and* $p_* > 0$, $(x_*, p) \sim (x, p_*)$.

Proof. By $*$, $(x, p_* - \varepsilon) < (x_*, p)$ for all ε small enough. By the continuity of $\gtrsim$, $(x, p_*) \lesssim (x_*, p)$. By a similar argument $(x_*, p) \lesssim (x, p_*)$. See the following diagram.



Proof of (b). Assume $(\sim_x, \sim_p, \gtrsim) \in C$. Assume $g(p_1)\,u(x_1) > g(p_2)\,u(x_2)$. Notice that $x_1 \neq 0$ and $p_1 \neq 0$. At least one of the inequalities $p_1 > p_2$ and $x_1 > x_2$ is true. If both are true, then by the monotonicity of $\gtrsim$, $(x_1, p_1) > (x_2, p_2)$. Assume $p_1 > p_2$ and $x_2 > x_1$. Define a sequence (x_1^k, p_1^k) as follows. First, $x_1^0 = x_1$ and $p_1^0 = p_1$. Then $x_1^k = (x_1^{k-1})^*$ and $p_1^k = (p_1^{k-1})_*$. By Lemma 2, $(x_1^k, p_1^k) \sim (x_1^{k-1}, p_1^{k-1})$. The functions u and g must satisfy $u(x_1^k) = u(x_1)\,\lambda^k$ and $g(p_1^k) = g(p_1)\,\lambda^{-k}$. Let K be the first such that $u(x_1^K) \geqslant u(x_{2*})$, $u(x_1^K) \leqslant u(x_2)$ and therefore

$$g(p_1^K)\,\lambda^k = g(p_1) > \frac{g(p_2)\,u(x_2)}{u(x_1)} \geqslant \frac{g(p_2)\,u(x_1^K)}{u(x_1)} = g(p_2)\,\lambda^K.$$

Thus $g(p_1^K) > g(p_2)$. Obviously, $p_1^K \not\sim p_{2*}$ and $x_1^K \sim_x x_{2*}$. Since $\gtrsim$ satisfies $*$ relative to $\sim_x$ and $\sim_p$ then $(x_1^K, p_1^K) > (x_{2*}, p_{2*})$. Thus $(x_1, p_1) \sim (x_1^K, p_1^K) > (x_{2*}, p_{2*})$.

Remark. Kevin Roberts drew my attention to the close relation between the above proof and the work of Y. K. Ng, see [7].

4. The Similarities Which Are Consistent with a Preference

In the previous section we asked the question what are the preferences which are consistent with $*$ relative to a given pair of similarities? In this section we ask the opposite question: Given a preference $\gtrsim$ what are the $\sim_x$ and $\sim_p$ satisfying $(\sim_x, \sim_p, \gtrsim) \in C$? The answer to this question provides a test for the plausibility of assumptions about preference relations.

Proposition 3. *Assume $\gtrsim$ is represented by the utility function $g(p)\,u(x)$, where g and u are positive, continuous, and strictly increasing functions. If $(\sim_x, \sim_p, \gtrsim) \in C$ then there is an $\lambda > 1$ such that*

$$x_1 \sim_x x_2 \quad \textit{iff} \quad 1/\lambda \leqslant u(x_2)/u(x_1) \leqslant \lambda$$

and

$$p_1 \sim_p p_2 \quad \textit{iff} \quad 1/\lambda \leqslant g(p_2)/g(p_1) \leqslant \lambda.$$

Proof. Pick $p_1 > p_2 \neq 0$ satisfying $p_1 \sim_p p_2$. For all $x_1 > x_2$, satisfying $x_1 \sim_x x_2$, $g(p_1)\,u(x_2) > g(p_2)\,u(x_1)$ and thus $g(p_1)/g(p_2) > u(x_1)/u(x_2)$. Therefore the set $\{u(x_1)/u(x_2) \mid x_1 \sim_x x_2\}$ is bounded and has a supremum λ_x. Similarly, let $\lambda_p = \sup\{g(p_1)/g(p_2) \mid p_1 \sim_p p_2\}$. Next it is shown that $\lambda_x = \lambda_p$. If $\lambda_x > \lambda_p$ we could have chosen $x_1 \sim_x x_2$ and $p_1 \nsim_p p_2$ such that $\lambda_x > u(x_1)/u(x_2) > g(p_2)/g(p_1) > \lambda_p$. By $*$, $(x_2, p_2) > (x_1, p_1)$ despite the fact that $g(p_1)\,u(x_1) > g(p_2)\,u(x_2)$.

Denote $\lambda = \lambda_x = \lambda_p$, and let $x_2 > x_1$, satisfying $\lambda > u(x_2)/u(x_1)$ and $x_1 \nsim_x x_2$. There are p_1 and p_2 satisfying $p_1 \sim_p p_2$ and $g(p_2)/g(p_1) > u(x_2)/u(x_1)$ and then by $*$, $(p_1, x_2) > (p_2, x_1)$ although $g(p_2)\,u(x_1) > g(p_1)\,u(x_2)$. Thus for all $x_2 > x_1$ such that $\lambda > u(x_2)/u(x_1)$, $x_1 \sim x_2$. By continuinity of $\sim_x$ it is also true that for all x_1 and x_2 satisfying $u(x_2)/u(x_1) = \lambda$, $x_1 \sim_x x_2$.

Conclusion. If $\gtrsim$ is represented by $pu(x)$ (or $g(p)x$) and $(\sim_x, \sim_p, \gtrsim) \in C$ then $\sim_p$ (or $\sim_x$) is a ratio similarity.

Thus to be consistent with expected utility theory a similarity on the probability dimension must be of the ratio similarity type. A possible explanation for the Allais paradox is that the underlying similarity relation for the probability space is not of the ratio type. Although $0.8/1 = 0.2/0.25$, the probabilities 0.2 and 0.25 are perceived to be similar while the probabilities 0.8 and 1 are not.

Only a ratio similarity on the price dimension is consistent with Yaari's dual theory (see Yaari [10]).

5. Final Remarks

Admittedly this paper was initiated as a reaction to the flow of papers which attempt to save utility theory paradigm by relaxing or modifying some of the VNM axioms. My feeling is that such attempts overlook the real objective of constructing a descriptive theory for decisions under risk. In order to construct such a theory one cannot avoid tackling the black box of the decision procedures themselves. This led me to formalize a certain property of decision-making procedures and to ascertain some of its consequences.

The results may have two interpretations. On the positive side it connects the notion of similarity with the notion of preference with a tight correspondence. On the negative side it casts doubt as to whether human choices from among a set of lotteries are indeed transitive. Property $*$ is only a first stage in the decision procedure. The property does not specify the way in which the preference over lotteries is determined in case $x_1 \not\sim_x x_2$ and $p_1 \not\sim_p p_2$. It is hard to believe that the rest of the procedure which people use is consistent with calculating the functions g and u of Proposition 2.

My own conclusion from this study is to quote again Herbert Simon: "There is an urgent need to expand the established body of economics analysis... to encompass the procedural aspects of decision-making."

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