Econometrica, Vol. 53, No. 5 (September, 1985)

A BARGAINING MODEL WITH INCOMPLETE INFORMATION ABOUT TIME PREFERENCES

BY ARIEL RUBINSTEIN[1]

The paper studies a strategic sequential bargaining game with incomplete information: Two players have to reach an agreement on the partition of a pie. Each player, in turn, has to make a proposal on how the pie should be divided. After one player has made an offer, the other must decide either to accept it or to reject it and continue the bargaining. Player 2 is one of two types, and player 1 does not know what type player 2 actually is.

A class of sequential equilibria (called bargaining sequential equilibria) is characterized for this game. The main theorem proves the (typical) uniqueness of the bargaining sequential equilibrium. It specifies a clear connection between the equilibrium and player 1's initial belief about his opponent's type.

1. INTRODUCTION

ONE OF THE MOST BASIC HUMAN SITUATIONS is one in which two individuals have to reach an agreement, chosen from among several possibilities. Traditional bargaining theory seeks to indicate one of the agreements as the expected (or desired) outcome on the sole basis of the set of possible agreements and on the point of non-agreement. Usually, the solution is characterized by a set of axioms. (For a survey of the axiomatic models of bargaining, see Roth [**13**].)

Much of the recent work on bargaining aims at explaining the outcome of a bargaining situation using additional information about the time preference of the parties and the bargaining procedure. Such models are associated with the strategic approach. The players' negotiating maneuvers are moves in a noncooperative game that describes the procedure of the bargaining; noncooperative solutions to the game are explored.

The strategic approach also seeks to combine axiomatic cooperative solutions and noncooperative solutions. Roger Myerson recently named this task the "Nash Program." Though Nash is usually associated with the axiomatic approach, he was the first to suggest that this approach must be complemented by a noncooperative game (see Nash [**10**]).

In a previous paper [**14**], I analyzed the following bargaining model, using the strategic approach. Two players have to reach an agreement on the partition of a pie of size 1. Each player in turn has to make a proposal on the division of the pie. After one player has made an offer, the other must decide either to accept it or to reject it and continue the bargaining. The players have preference relations which are defined on the set of ordered pairs (s, t), which is interpreted as agreement on partition s at time t. Several properties are assumed: "pie" is

[1] This research began when I was a research fellow at Nuffield College, Oxford. It was continued at Bell Laboratories and at the Institute for Advanced Studies of the Hebrew University. I am grateful to these three institutions for their hospitality.

I owe thanks to many friends with whom I have discussed this subject during the past five years. Specifically I would like to thank Asher Wolinsky, Leo Simon and Ed Green for many valuable remarks; Avner Shaked, whose idea helped me simplify the proof of the main theorem; and an associated editor and referee of this journal for their extraordinarily helpful remarks.

deairable, "time" is valuable, the preference is continuous and stationary, and the larger the portion of the pie the more "compensation" a player needs to consider a delay of one period immaterial.

The set of outcomes of Nash equilibria for this game includes almost every possible agreement, and the bargaining in a Nash equilibrium may last beyond the first round of negotiations. The Nash equilibria are protected by threats, such as responding to a deviation by insisting on receiving the whole pie. It is very useful to apply the concept of (subgame) perfect equilibrium (see Selten [**16**]) which requires that the players' strategies induce an equilibrium in any subgame. This usually leads to a single solution, which is characterized by a pair of partitions, P_1 and P_2, that satisfies (i) Player 1 is indifferent between "P_2 today" and "P_1 tomorrow," and (ii) Player 2 is indifferent between "P_1 today" and "P_2 tomorrow." When a unique pair of P_1 and P_2 satisfies the above statements, the only perfect equilibrium partition is P_1 when 1 starts the bargaining and P_2 when 2 starts the bargaining. The structure of the unique perfect equilibrium is as follows: Player 1 (2) always suggests P_1 (P_2) and Player 2 (1) accepts any offer which is better for him than P_1 (P_2). For example, when each player has a fixed discounting factor δ, the perfect equilibrium outcome gives player 1 $1/(1+\delta)$ if he starts the negotiation, and $\delta/(1+\delta)$ if 2 starts. Both partitions tend to the equal partition when δ tends to 1. It is clear that the method demonstrated by this model is useful for analyzing other bargaining procedures and other properties of time preferences. Additional properties of the model are studied by Binmore [**1**].

A critical assumption of the model is that each player has complete information about the other's preference. This assumption makes it less surprising that, typically, the bargaining in perfect equilibrium ends in the first period, although it could continue endlessly.

When incomplete information exists, new elements appear; a player may try to conclude from the other player's moves who his opponent really is; the other player may try to cheat him by leading him to believe that he is tougher than he actually is. In general terms, the incomplete information model enables us to address the issues of "reputation building," signalling, and self-selection mechanisms. Here continuation of bargaining beyond the first period becomes more likely.

A number of works have appeared on bargaining with incomplete information. For example, Harsanyi and Selten [**7**] present a generalized Nash solution for two-person bargaining games with incomplete information. Myerson [**9**] presents another generalization. Both solutions are characterized by sets of axioms. Finite-horizon bargaining games with incomplete information are treated by Fudenberg and Tirole [**5**], Sobel and Takahashi [**17**], and Ordover and Rubinstein [**11**]. In addition, Cramton [**2**], Perry [**12**], and Fudenberg, Levine, and Tirole [**4**] analyze seller-buyer infinite-horizon bargaining games in which reservation prices are uncertain, but time preferences are known.

The present article attempts to explain bargaining with incomplete information by investigating the model introduced in [**14**], with one additional element: Player

2 may be one of two types: 2_w (weak) or 2_s (strong). The types differ in their time-preferences (for example, higher or lower discounting factors). Player 1 adopts an initial belief regarding the identity of Player 2.

The difficulties of extending the notion of perfect equilibrium to games with incomplete information have been discussed extensively in the last few years (see Kreps and Wilson [**8**]). Generally speaking, the problem is that in order to check the optimality of 1's strategy after a given sequence of moves, we must verify his beliefs. Therefore, the notion of sequential equilibrium includes player 1's beliefs both on and off the equilibrium path.

The set of sequential equilibria for this game is very large. The freedom to choose new conjectures off the equilibrium path enables the players to establish credibility for "too many" threats and thus to support "too many" equilibria. I suggest that many of the sequential equilibria are unreasonable. These can be eliminated by making additional requirements on beliefs and on equilibrium behavior. Indeed, as Kreps–Wilson claims, "... the formation [of sequential equilibria] in terms of players' beliefs gives the analyst a tool for choosing among sequential equilibria" [**8**, p. 884].

The quite reasonable, additional requirements are described in Section 5. The key requirement $(B-1)$ makes it possible for type 2_s to screen himself: if player 2 rejects an offer made by player 1 and makes a counteroffer which is worse than 1's offer for the weak type but better for the strong type, then 1 concludes that player 2 is strong (2_s).

The main theorem of this paper specifies a clear connection between the unique bargaining sequential equilibrium and 1's initial belief that 2 is 2_w, denoted by ω_0. The theorem states that there exists a cut-off point ω^* such that if ω_0 is strictly below ω^*, player 1 gives up: he offers the partition he would have offered if he thought he was playing against 2_s, and player 2, whatever his type, accepts it. If ω_0 is strictly above ω^*, some continuation of the bargaining is possible. In equilibrium, player 1 offers P_1, 2_w would accept it, and 2_s would reject it and offer P_2, which is accepted by 1. The partitions P_1 and P_2 are the only ones satisfying: (i) Player 1 is indifferent between "P_2 today" and the lottery of "P_1 tomorrow" with probability ω_0 and "P_2 after tomorrow" with probability $1-\omega_0$; (ii) Player 2_w is indifferent between "P_1 today" and "P_2 tomorrow".

2. THE BARGAINING MODEL

Two players, 1 and 2, are bargaining on the partition of a pie. The pie will be partitioned only after the players reach an agreement. Each player in turn makes an offer and his opponent may agree to the offer, "Y", or reject it, "N". Acceptance of an offer ends the bargaining. After rejection, the rejecting player has to make a counteroffer and so on without any given limit. There are no rules which bind the players to previous offers they have made.

Formally, let $S=[0,1]$. A partition of the pie is identified with a number s in S by interpreting s as the proportion of the pie that 1 receives (2 receives $1-s$).

A strategy specifies the offer that a player makes whenever it is his turn to make an offer, and his reaction to any offer made by his opponent. A strategy includes the player's plans even after a series of moves that are inconsistent with the strategy itself.

Let F denote the set of all strategies available to the player who starts the bargaining. Formally, F is the set of all sequences of functions $f=\{f^t\}_{t=0}^{\infty}$, when for t even, $f^t\colon S^{t-1}\to S$ and for t odd, $f^t\colon S^t\to\{Y, N\}$, where S^t is the set of all sequences of length t of elements of S. (For example, $f^2(s^0, s^1)$ is a player's offer at time 2 assuming that he offered s^0, his opponent rejected it and made the offer s^1, which was rejected by the other player.) Similarly, G is the set of all strategies for a player whose first move is a response to the other player's offer. Note that mixed strategies are not allowed.

A typical outcome of the game is a pair (s, t), which is interpreted as agreement on the partition s in period t. Perpetual disagreement is denoted by $(0, \infty)$.

The outcome function of the game $P(f, g)$, then, takes the value (s, t) if two players who adopt strategies f and g reach an agreement s at period t, and the value $(0, \infty)$ if they do not reach an agreement.

The players have preference relations $\succcurlyeq_1$ and $\succcurlyeq_2$ on the set of pairs $S\times T=[0,1]\times\{0,1,2,\dots\}$. It is assumed that $\succcurlyeq_1$ satisfies the following assumptions (analogous assumptions are assumed about $\succcurlyeq_2$; recall that according to our notation, player 2 receives the fraction $1-x$ in the partition x):

ASSUMPTION (A-0): *The relation is complete, reflexive, and transitive.*

ASSUMPTION (A-1): "*Pie*" *is desirable*: *If* $x>y$, *then* $(x, t)\succ_1(y, t)$.

ASSUMPTION (A-2): "*Time*" *is valuable*: *If* $x>0$ *and* $t_2>t_1$, *then* $(x, t_1)\succ_1(x, t_2)$.

ASSUMPTION (A-3): *Continuity*: *The graph of* $\succcurlyeq_1$ *is closed in* $(S\times T)\times(S\times T)$.

ASSUMPTION (A-4): *Stationarity*: $(x, t)\succcurlyeq_1(y, t+1)$ *iff* $(x, 0)\succcurlyeq_1(y, 1)$.

ASSUMPTION (A-5): *The increasing compensation property*: *If*

$$(x, 0)\sim_1(x+\varepsilon(x), 1),$$

then ε *is strictly increasing.*

For the sake of simplicity I make the following assumption:

ASSUMPTION (A-6): *For every* x *there is* $d(x)\in[0,1]$, *such that* $(x, 1)\sim_1(d(x), 0)$.

This implies that $(0, t)\sim_1(0, 0)$ for all t and I assume further that $(0, \infty)\sim_1(0, 0)$.

The present value of (x, t) is the t-fold composition of d with itself. Notice that by (A-1)-(A-3) and (A-6) the function d is increasing, continuous, and satisfies $d(0)=0$ and $d(x)<x$ for $x>0$. By Assumption (A-5), $x-d(x)$ is an increasing function and by Assumption (A-4) the function d gives us all the information about the relation $\succcurlyeq_1$.

The most important family of relations satisfying the above assumptions are those induced by a utility function $x\delta^t$. The number $0<\delta<1$ is interpreted as the fixed discounting factor.

It is shown in Fishburn and Rubinstein [**3**] that time preferences satisfying (A-0)-(A-4) and (A-6) can be represented by a utility function of the form $u(x)\delta^t$. A sufficient (but not necessary) condition for the preference to satisfy Assumption (A-5) is that it be representable by $u(x)\delta^t$ where u is concave. The fixed bargaining costs preference (induced from a utility function $x-ct$) does not satisfy Assumptions (A-5) and (A-6).

In the current paper player 1's preference must be extended to refer to the lotteries of elements in $S\times T$. (The symbol $\omega 0_1 \oplus (1-\omega)0_2$ stands for the lottery which provides the outcome 0_1 with probability ω, and outcome 0_2 with probability $(1-\omega)$.)

ASSUMPTION (A-7): *Player maximizes the expectation of* $u(x)\delta^t$ *for some concave function* $u(x)$ *and some* δ.

The new element in this paper is the extension of the analysis to situations of incomplete information. Player 1's preference is known by player 2, but player 2 may possess one of the preferences $\succcurlyeq_w$ (weak) or $\succcurlyeq_s$ (strong). If 2 holds $\succcurlyeq_w$ ($\succcurlyeq_s$) it is said that he is of type 2_w (2_s).

It is assumed that 2_w is more impatient than 2_s, that is:

ASSUMPTION (C-1): *If* $x\neq 1$ *and* $(y, 1)\sim_w (x, 0)$, *then* $(y, 1)\succ_s (x, 0)$.

With fixed discounting factors δ_w and δ_s, this assumption means that $\delta_s>\delta_w$.

In [**14**] a full characterization of the perfect equilibrium outcomes of this game with complete information is presented. We summarize these results here.

Under the current assumptions about the time preferences there is a unique $(x^*, y^*)\in S^2$ satisfying

$$(x^*, 1)\sim_1 (y^*, 0) \quad \text{and} \quad (y^*, 1)\sim_2 (x^*, 0).$$

Denote the pair (x^*, y^*) by $\Delta(\succcurlyeq_1, \succcurlyeq_2)$. It was proved in [**14**] that the only perfect equilibrium partition is x^* when 1 starts the bargaining, and y^* when 2 starts the bargaining and in any case the negotiation ends in the first period. Denote

$$\Delta(\succcurlyeq_1, \succcurlyeq_w)=(V_w, \hat{V}_w),$$
$$\Delta(\succcurlyeq_1, \succcurlyeq_s)=(V_s, \hat{V}_s).$$

Since 2_w is more impatient than 2_s, $V_w>V_s$ and $\hat{V}_w>\hat{V}_s$.

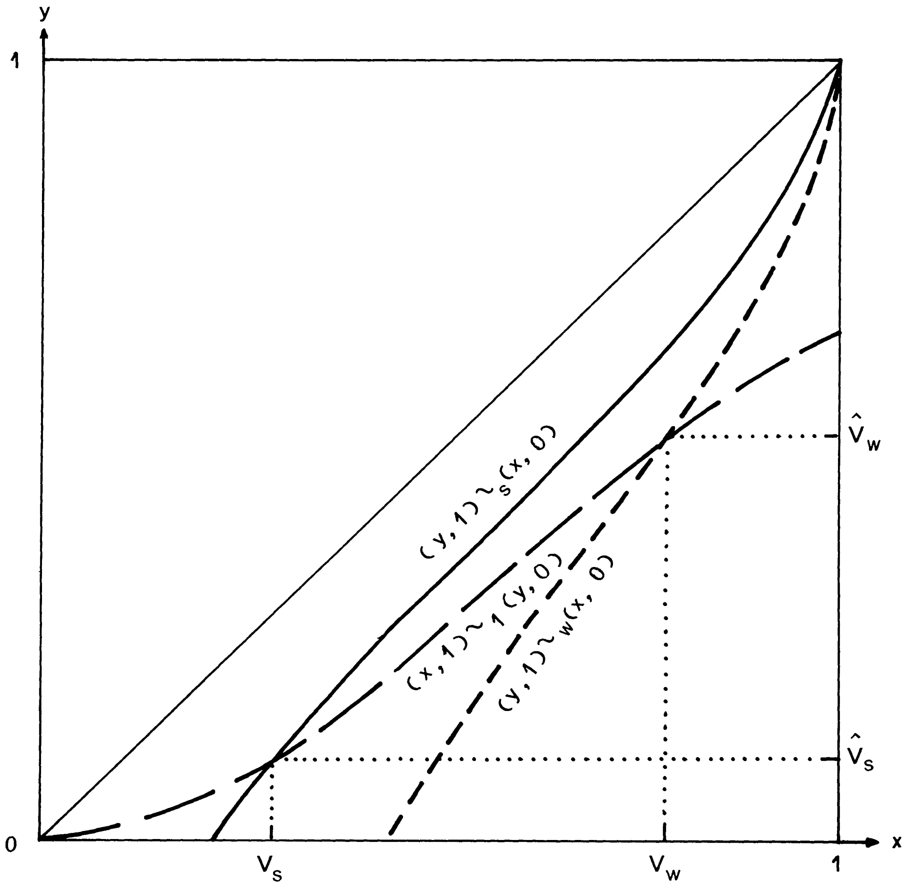

FIGURE 1

Figure 1 is useful at several subsequent points. The curve which includes the origin represents the present value to player 1 of the partition x tomorrow, that is, it includes all the pairs (x, y) such that $(x, 1) \sim_1 (y, 0)$. The other two curves represent the present value to player 2 of y tomorrow, depending on whether he is weak or strong. That is, they include the pairs (x, y) such that $(y, 1) \sim_s (x, 0)$ and $(y, 1) \sim_w (x, 0)$. The intersections of those curves with player 1's present value curve are $(V_s, \hat{V}_s)$ and $(V_w, \hat{V}_w)$.

EXAMPLE: When the players have fixed discounting factors δ_1 and δ_2,

$$\Delta = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \delta_1 \frac{1-\delta_2}{1-\delta_1\delta_2}\right).$$

Binmore [1] shows that where the δ_i are derived from the continuous discounting formula $x(e^{-r_i\tau})^t$ (τ is the length of one period of bargaining), then

$$\lim_{\tau\to 0} \Delta = \left(\frac{r_2}{r_1+r_2}, \frac{r_2}{r_1+r_2}\right).$$

In other words, when the time interval tends to 0, player 1 gets r_2/r_1+r_2, regardless of whether he is the first or the second player to make an offer. In particular, if the players have identical time preferences, the solution coincides with the equal partition.

The last assumption about the time preferences is the following:

ASSUMPTION (C-2): $(\hat{V}_w, 0) <_w (V_s, 1)$.

By (C-1), $(\hat{V}_w, 0) <_w (\hat{V}_s, 0)$. Here it is further assumed that type 2_w prefers the complete information partition between 1 and 2_s even if player 1 starts the bargaining, and there is a delay of one period in the agreement. Assumption (C-2) excludes the possibility that in equilibrium 2_w sorts himself by making an offer z satisfying $(z, 0) \geqslant_1 (V_w, 1)$ (and thus $(z, 0) \leqslant_w (\hat{V}_w, 0)$) and $(z, 0) \geqslant_w (V_s, 1)$ (see Proposition 4). Notice that if the players have discounting factors $\delta_i = e^{-r_i\tau}$ (see the previous example), then Assumption (C-2) is satisfied whenever $r_w > r_s$ and τ is small enough.

3. NASH EQUILIBRIA

Section 2 defines a game with incomplete information. The usual solution concept for such games is Harsanyi's Bayesian Nash equilibrium (see [6]). Let (f, g, h) be a triple of strategies for 1, 2_w, and 2_s respectively. The outcome of the play of (f, g, h) is

$$P(f, g, h) = \langle P(f, g), P(f, h)\rangle.$$

This means that the outcome of the game is a pair of outcomes, one each for the cases that 2 is actually 2_w or 2_s.

DEFINITION: A triple $(\hat{f}, \hat{g}, \hat{h}) \in F\times G\times G$ is a Nash equilibrium if there is no $f\in F$ such that

$$\omega_0 P(f, \hat{g}) \oplus (1-\omega_0) P(f, \hat{h}) >_1 \omega_0 P(\hat{f}, \hat{g}) \oplus (1-\omega_0) P(\hat{f}, \hat{h}),$$

and there is no $g\in G$ or $h\in G$ such that

$$P(\hat{f}, g) >_w P(\hat{f}, \hat{g}) \quad \text{or} \quad P(\hat{f}, h) >_s P(\hat{f}, \hat{h}).$$

(A similar definition is suitable for $(\hat{f}, \hat{g}, \hat{h}) \in G\times F\times F$.)

In other words, 1's strategy has to be a best response against 2_w's and 2_s's plans, "weighted" by ω_0 and $1-\omega_0$, respectively, and $\hat{g}$ and $\hat{h}$ have to be best responses in the usual sense.

As in the complete information case, the set of Nash equilibria in this model is very large. Proposition 1 provides a complete characterization of the set of Nash equilibrium outcomes in this model. In particular, Proposition 1 implies that, for every partition P, $\langle(P, 0), (P, 0)\rangle$ is a Nash equilibrium outcome.

PROPOSITION 1: $\langle(P_w, t_w), (P_s, t_s)\rangle = \langle 0_w, 0_s\rangle$ *is a Nash equilibrium outcome if and only if*

$$0_w \succcurlyeq_w 0_s \quad \textit{and} \quad 0_s \succcurlyeq_s 0_w.$$

PROOF: If $\langle 0_w, 0_s\rangle$ satisfies the conditions, then the following is a description of a Nash equilibrium: both players demand the entire pie and reject every offer except at periods t_w and t_s when players 1 and 2_w, and 1 and 2_s, offer and accept P_w and P_s, respectively.

Obviously the conditions $0_w \succcurlyeq_w 0_s$ and $0_s \succcurlyeq_s 0_w$ are necessary. *Q.E.D.*

4. SEQUENTIAL EQUILIBRIUM

The basic idea of sequential equilibrium is similar to the idea of perfect equilibrium: the players' strategies are best responses not only at the starting point, but at any decision node. For player 1, the test whether a strategy is the best response depends on 1's belief that 2 is 2_w. Therefore, a sequential equilibrium includes the method of updating 1's belief that 2 is 2_w.

Define a belief system to be a sequence $\boldsymbol{\omega} = (\omega^t)_{t=-1,1,3,\ldots}$ such that $\omega^{-1} = \omega_0$ and, for $t \geq 1$, $\omega^t: S^t \to [0, 1]$. $\omega^t(s^1, \ldots, s^t)$ is 2's belief that 2 is 2_w, after the sequence of rejected offers $s^1, \ldots, s^{t-1}$ and after 2 made offer s^t.

The formal definition of $(f, g, h, \boldsymbol{\omega})$ being a sequential equilibrium is messy but intuitively straightforward. The belief system must be consistent with the Bayesian formula. Moreover, after unexpected behavior by player 2, player 1 makes a new conjecture regarding 2's type; the equilibrium behavior of player 1 must be consistent with the new conjecture as long as no new deviation is observed. Player 1 does not change his belief about player 2 as a result of any deviation of his own.

For formal definition, let us use the notation $\mathbf{s}^t = (s^0 \cdots s^t)$ and the brief notation $\omega^t = \omega^t(\mathbf{s}^t)$, $f^t = f^t(\mathbf{s}^t)$, or $f^t(\mathbf{s}^{t-1})$ according to the evenness of t, and similarly define g^t and h^t.

The Bayesian requirements are:

(i) If $g^{T-1} = h^{T-1} = N$ and $g^T = h^T = s^T$, then $\omega^T = \omega^{T-2}$.

(ii) If $0 < \omega^{T-2} < 1$, $g^{T-1} = N$ and $g^T = s^T$, and either $h^{T-1} = Y$ or $h^T \neq s^T$, then $\omega^T = 1$.

(iii) If $0 < \omega^{T-2} < 1$, $h^{T-1} = N$, $h^T = s^T$ and either $g^{T-1} = Y$ or $g^T \neq s^T$, then $\omega^T = 0$.

Finally I add to the definition of sequential equilibrium another constraint which does not follow from Kreps-Wilson's definition. If 1 concludes with probability 1 that player 2 is of a certain type, he continues to hold this belief whatever occurs after he comes to this conclusion; in other words, if he concludes that $\omega^t = 0$ or $\omega^t = 1$, he continues to play the game as if it were a game with complete information against 2_s or 2_w respectively.

Formally, if $\omega^{T-2} = 1$, then $\omega^T = 1$, and if $\omega^{T-2} = 0$, then $\omega^T = 0$.

The next proposition can be proved using the arguments presented in [**14**].

PROPOSITION 2: *Let $(f, g, h, \boldsymbol{\omega})$ be a sequential equilibrium in the game when 1 starts; then $P(f, h) \succcurlyeq_1 (V_s, 0)$ and $P(f, g) \preccurlyeq_1 (V_w, 0)$; and if 2 starts, $P(f, h) \succcurlyeq_1 (\hat{V}_s, 0)$ and $P(f, g) \preccurlyeq_1 (\hat{V}_w, 0)$.*

Thus, a sequential equilibrium outcome cannot be better (worse) for player 1 than the perfect equilibrium outcome in the complete information bargaining game with 2_w (2_s).

The next proposition demonstrates that the set of sequential equilibrium outcomes is very large.

The sequential equilibrium, which is constructed in the proof, is supported by player 1's belief that a deviation must be of type 2_w. Whenever there is a deviation from the equilibrium path, player 1 concludes that he is playing against 2_w. Such conjectures serve as threats against player 2. The definition of sequential equilibrium that was suggested to exclude noncredible threats does not exclude threatening by beliefs.

PROPOSITION 3: *For every ω_0, and for all x^* satisfying $(\hat{V}_s, 1) \succcurlyeq_w (x^*, 0)$ and $(x^*, 0) \succcurlyeq_s (\hat{V}_w, 1)$, either $\langle (x^*, 0), (x^*, 0)\rangle$ or $\langle (x^*, 0), (y^*, 1)\rangle$ (where $(y^*, 1) \sim_w (x^*, 0)$) is a sequential equilibrium outcome.*

PROOF: Let x^* satisfy $(\hat{V}_s, 1) \succcurlyeq_w (x^*, 0)$ and $(x^*, 0) \succcurlyeq_s (\hat{V}_w, 1)$. Let y^* and z^* satisfy $(y^*, 1) \sim_w (x^*, 0)$ and $(z^*, 0) \sim_s (y^*, 1)$. Let $\tilde{y}$ and $\tilde{z}$ satisfy $(\tilde{y}, 0) \sim_1 (x^*, 1)$ and $(\tilde{z}, 0) \sim_w (\tilde{y}, 1)$. Let us distinguish between the following two comprehensive (and nonexclusive) cases:

CASE I: $\omega_0(x^*, 0) \oplus (1-\omega_0)(y^*, 1) \succcurlyeq_1 (z^*, 0)$.

CASE II: $\omega_0(\tilde{z}, 0) \oplus (1-\omega_0)(\tilde{y}, 1) \preccurlyeq_1 (x^*, 0)$.

By Proposition 7, at least one of the following inequalities is true:

$$\omega_0(x^*, 1) \oplus (1-\omega_0)(y^*, 2) \succcurlyeq_1 (y^*, 0),$$

$$(\tilde{y}, 0) \succcurlyeq_1 \omega_0(\tilde{z}, 1) \oplus (1-\omega_0)(\tilde{y}, 2).$$

Combining $(y^*, 0) \succcurlyeq_1 (z^*, 1)$ and $(x^*, 1) \sim_1 (\tilde{y}, 0)$ with the stationarity of $\succcurlyeq_1$, we get that for all ω_0 either Case I or Case II must be true.

The following is a description of a sequential equilibrium for Case I with the outcome $\langle (x^*, 0), (y^*, 1)\rangle$:

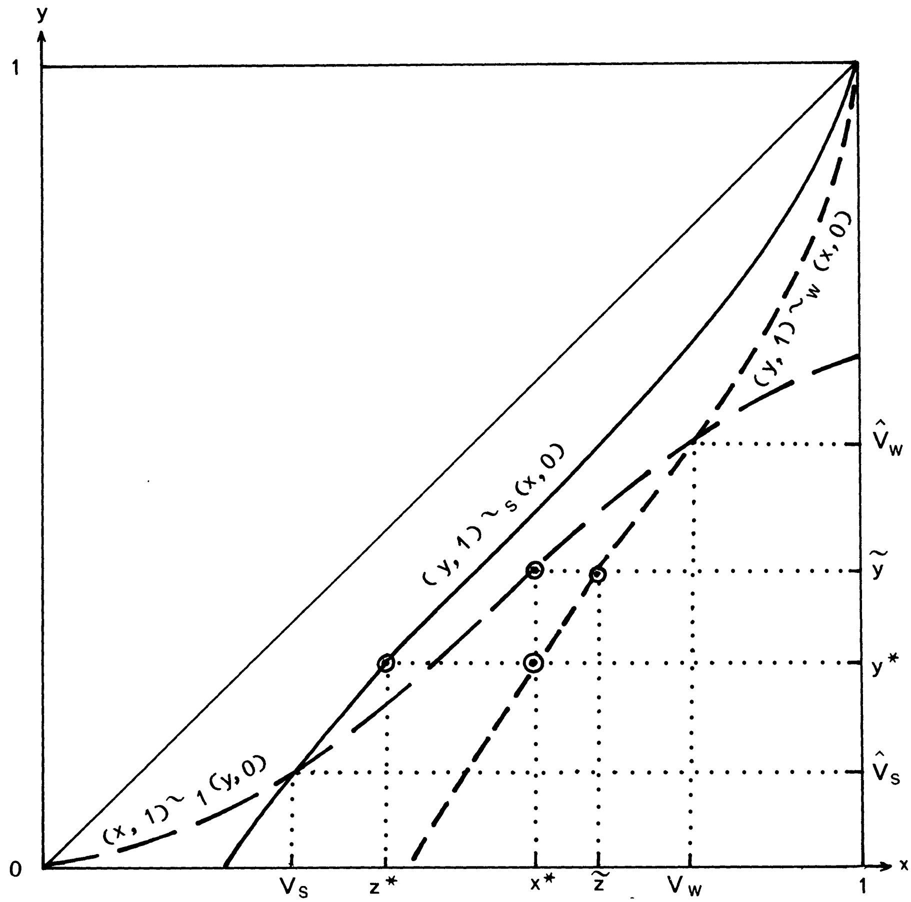

FIGURE 2

Unless 1 changes his belief, he always demands x^* and agrees at most to y^*; type 2_w accepts only any offer below x^* and demands y^*; type 2_s accepts offer x satisfying $(x, 0) \succcurlyeq_s (y^*, 1)$ and always offers y^*. On condition that 2 made a move unplanned by any of the types, 1 changes ω^t to 1 and the continuation is as in the complete information game between 1 and 2_w. Clearly the only change for a profitable deviation of 1 would be to make a lower demand like z^* which would be accepted by both types. But since $\omega_0(x^*, 0) \oplus (1-\omega_0)(y^*, 1) \succcurlyeq_1 (z^*, 0)$, this is not profitable. Checking the other conditions of the sequential equilibrium condition is straightforward.

The following is a description of a sequential equilibrium for case II with the outcome $\langle (x^*, 0), (x^*, 0) \rangle$:

Unless 1 changes his belief, he always offers x^* and accepts y only if $(y, 0) \succcurlyeq_1 (x^*, 1)$, $(y \geqslant \tilde{y})$. Both 2_w and 2_s agree to settle for x only if $(x, 0)$ is preferable (according to $\succcurlyeq_w$ and $\succcurlyeq_s$ respectively) to $(\tilde{y}, 1)$. They always offer $\tilde{y}$. After 2 made a move unexpected from both types, 1 changes ω^t to 1 and the continuation is as in the complete information game between 1 and 2_w.

Notice that if 1 demands $x > x^*$, 2_s rejects it because $(\tilde{y}, 1) \succ_s (x^*, 0)$. Thus the most that 1 can achieve by deviating is that 2_w would agree to $\tilde{z}$ and 2_s would offer $\tilde{y}$. However since $\omega_0(\tilde{z}, 0) \oplus (1-\omega_0)(\tilde{y}, 1) \preccurlyeq_1 (x^*, 0)$, this is not profitable. The requirement that $(x^*, 0) \succcurlyeq_s (\hat{V}_w, 1)$ is important to assure that 2_s will not gain by rejecting x^*. *Q.E.D.*

5. BARGAINING SEQUENTIAL EQUILIBRIUM

Section 4 shows that it might be desirable to place additional requirements on the belief systems in a sequential equilibrium $(f, g, h, \boldsymbol{\omega})$.

In order to demonstrate the first Assumption, (B-1), imagine that 1 makes an offer x and 2 rejects it and offers y which satisfies that $(y, 1) \succcurlyeq_s (x, 0)$ and $(x, 0) \succ_w (y, 1)$. That is, 2 presents a counteroffer which is better for 2_s and worse for 2_w than the original offer x. Then, we assume, 1 concludes that he is playing against 2_s.

This assumption is related to an element which is missing from most studies in Game Theory: we tend to conclude facts from other people's behavior even when the unexpected occurs. (B-1) is this type of inference. The effect of (B-1) is to exclude sequential equilibria like those constructed in the proof of Proposition 3.

ASSUMPTION (B-1): *$\boldsymbol{\omega}$ is such that if $\omega^{t-2}(s^{t-2}) \neq 1$, $(s^t, 1) \succcurlyeq_s (s^{t-1}, 0)$, and $(s^{t-1}, 0) \succ_w (s^t, 1)$, then $\omega^t(s^1 \cdots s^{t-1}, s^t) = 0$.*

The next assumption states that 2's insistence cannot be an indication that 2 is more likely to be 2_w. Assume that 2 rejects an offer x and suggests an offer y, such that

$$(y, 1) \succcurlyeq_w (x, 0) \quad \text{and} \quad (y, 1) \succcurlyeq_s (x, 0).$$

Then when 1 updates his belief, his subjective probability that he is playing against 2_w does not increase.

ASSUMPTION (B-2): *If $(s^t, 1) \succcurlyeq_s (s^{t-1}, 0)$ and $(s^t, 1) \succcurlyeq_w (s^{t-1}, 0)$, then $\omega^t(s^t) \leqslant \omega^{t-2}(s^{t-2})$.*

The next two assumptions place direct restrictions on the players' equilibrium behavior, rather than describing their beliefs or preferences.

Assumption (B-3) is a "tie-breaking" assumption. If player 1 has been offered a partition x and after rejecting it he expects to reach an agreement whereby he is indifferent to x, then 1 accepts x.

ASSUMPTION (B-3): *If* $\langle P(f_{|s^1\cdots s^t}, g_{|s^1\cdots s^t}, h_{|s^1\cdots s^t}), 1\rangle \sim_1 (s^t, 0)$, *then* $f(s^t) = Y$. ($f_{|s^1\cdots s^t}$ *is the residual strategy of* f *after the history* $s^1 \cdots s^t$.)

The last assumption is that player 2 never makes an offer lower than $\hat{V}_s$.

ASSUMPTION (B-4): *Whenever it is* 2*'s turn to make an offer,*

$$g^t \geq \hat{V}_s \quad \text{and} \quad h^t \geq \hat{V}_s.$$

Notice that by Proposition 2, player 2 rejects any offer which is lower than $\hat{V}_s$. Still, the players could use such offers as a communication method. Assumption (B-4) excludes this possibility.

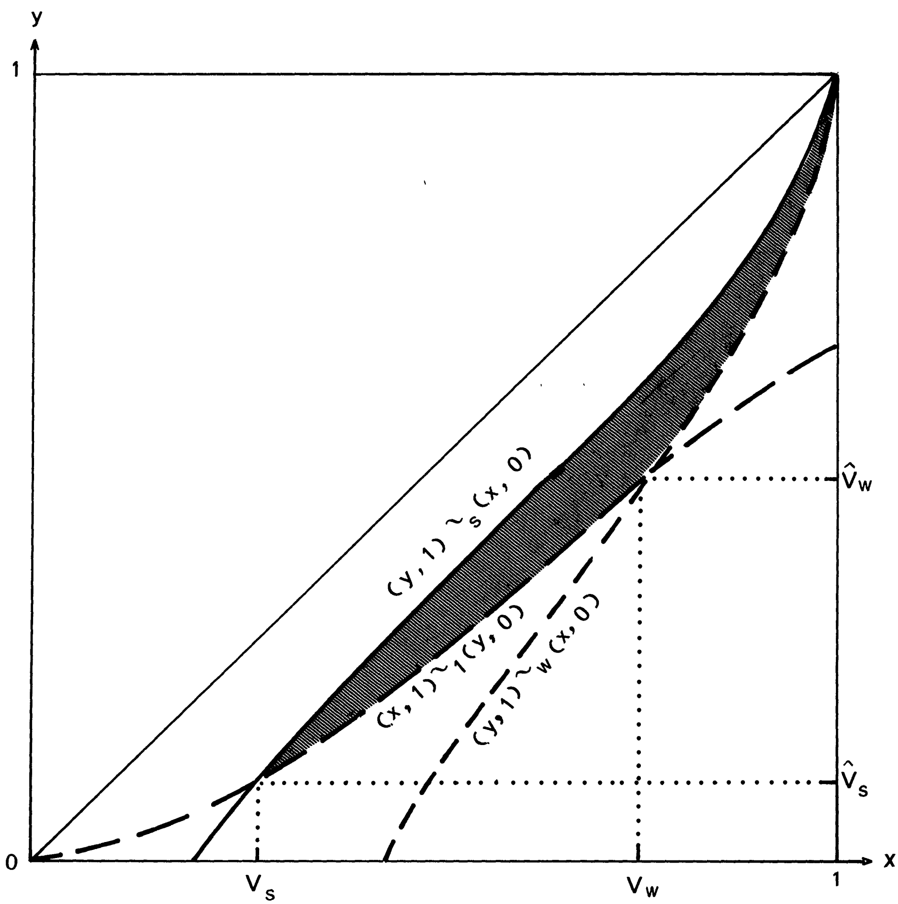

FIGURE 3

I would also like to mention the associate editor's suggestion that Assumption (B-4) can be replaced by a requirement on the beliefs that if 2_w is supposed to accept an offer and 2_s is supposed to reject it, then whatever is 2's offer, 1 concludes from 2's rejection that he is playing against 2_s. The reader can verify that the alternative assumption plays the same role as Assumption (B-4) in the only place it is used (the Proof of Proposition 6).

DEFINITION: (f, g, h, ω) is a *bargaining sequential equilibrium* if it is a sequential equilibrium and satisfies (B-1)–(B-4).

For the following examination of some of the properties of bargaining sequential equilibrium, let (f, g, h, ω) be a bargaining sequential equilibrium:

PROPOSITION 4: *Whenever it is 2's turn to make an offer, 2_w and 2_s make the same offer.*

PROOF: Assume that there is a history after which 2_w and 2_s make the offers y and z, respectively, and that $y \neq z$. If 1 accepts both y and z, then the type making the higher offer will deviate to the lower offer.

If 1 rejects both y and z, then in the next period he believes he knows which type 2 is, and he offers V_w or V_s accordingly. Then type 2_w will gain by offering z.

If 1 accepts only z, then the outcome against 2_w will be V_w in the next period. By definition of sequential equilibrium, $(z, 0) \succcurlyeq_s (V_w, 1)$. By Assumption (C-1), $(z, 0) \succ_w (V_w, 1)$. Thus, 2_w can deviate and gain by offering z.

If 1 accepts y and rejects z, it must be that $(y, 0) \succcurlyeq_1 (V_w, 1)$ and $(y, 0) \succcurlyeq_w (V_s, 1)$, in contradiction to Assumption (C-2). *Q.E.D.*

PROPOSITION 5: *If both 2_w and 2_s accept an offer x, then $x \leqslant V_s$.*

PROOF: If $x > V_s$, then 2_s may deviate, say "N" and suggest y at the next period, where $(y, 1) \succ_s (x, 0)$, $(y, 1) \prec_w (x, 0)$, and $(y, 0) \succ_1 (V_s, 1)$.

Any y that satisfies that (x, y) is in the shaded area in Figure 3 will do. By Assumption (B-1), player 1 concludes that 2 is 2_s, and since $(y, 0) \succ_1 (V_s, 1)$, he accepts the offer y; since $(y, 1) \succ_s (x, 0)$, 2_s gains by the deviation. When $x = 1$ there is no (x, y) in the shaded area of the figure. However, $(\hat{V}_w, 1) \succ_s (1, 0)$ and 2_s could gain by offering and accepting $\hat{V}_w$. *Q.E.D.*

PROPOSITION 6: *If 2_w accepts offer x and 2_s rejects it, then 2_s makes an offer y such that $(y, 1) \sim_w (x, 0)$, and player 1 accepts it.*

PROOF: If 1 rejects y then he offers V_s in the next period. Therefore, $(V_s, 1) \succcurlyeq_1 (y, 0)$, and by Assumption (B-3) $(V_s, 1) \succ_1 (y, 0)$ which means $y < \hat{V}_s$, contradicting (B-4). Thus, 1 accepts y, which implies $(y, 0) \succcurlyeq_1 (V_s, 1)$. It must be that $(x, 0) \succcurlyeq_w (y, 1)$. If $(x, 0) \succ_w (y, 1)$ and $y > \hat{V}_s$, then 2_s gains by decreasing the offer to $y - \varepsilon$ (for $\varepsilon > 0$ small enough). This lower offer persuades 1 that 2 is 2_s and 1 accepts it because $(y - \varepsilon, 0) \succ_1 (V_s, 1)$.

If $(x, 0) \succ_w (y, 1)$ and $y = \hat{V}_s$, then 1 may deviate and demand $x + \varepsilon$. Type 2_w will accept the offer if ε is small enough to satisfy $(x + \varepsilon) \succ_w (y, 1)$, and 2_s will reject it and will offer $\hat{V}_s$, the same offer he intended to make before the deviation. Thus 1 gains by the deviation. *Q.E.D.*

REMARK: Unless we assume (B-3) and (B-4) we can get additional equilibria, where 1 offers x, 2_w accepts x, 2_s rejects it and offers y $(y < \hat{V}_s)$, which 1 rejects in favor of the agreement V_s in the next period.

From Propositions 4, 5, and 6 we may conclude that a bargaining sequential equilibrium must end with one of the following:

(T-1): 1 *offers* x, 2_w *accepts it and* 2_s *rejects it*; 2_s *offers* y *at the next period and* 1 *accepts it. The offer* y *satisfies*

$$(x, 0) \sim_w (y, 1),$$

$$(y, 0) \succcurlyeq_1 (V_s, 1) \quad (i.e.,\ y \geqslant \hat{V}_s),$$

and

$$\omega_0(x, 0) \oplus (1 - \omega_0)(y, 1) \succcurlyeq_1 (V_s, 0).$$

(T-2): 1 *offers* V_s *and* 2 *accepts it.*

(T-3): 2_w *and* 2_s *offer* y *and* 1 *accepts it.*

6. THE POINT (x^ω, y^ω)

Let $z(x)$ be the z satisfying $(x, 0) \sim_w (z, 1)$ if such a z exists. Define for every $0 \leqslant \omega \leqslant 1$,

$$d^\omega(x) = y \quad \text{where} \quad (y, 0) \sim_1 \omega(x, 1) \oplus (1 - \omega)(z(x), 2).$$

Thus, $d^\omega(x)$ is the minimum that 1 would now agree to accept from 2 if he expects a bargaining sequential equilibrium in the subgame starting the next period, where his agreement with 2_w is x, and his agreement with 2_s is $z(x)$ one period later.

The function d^ω has several straightforward properties. It is continuous, it is strictly increasing, it satisfies $d^\omega(x) < d_1(x) < x$ for $x \geqslant V_s$, where $(d_1(x), 0) \sim_1 (x, 1)$. It satisfies $d^{\omega_1}(x) \geqslant d^{\omega_2}(x)$ if $\omega_1 > \omega_2$.

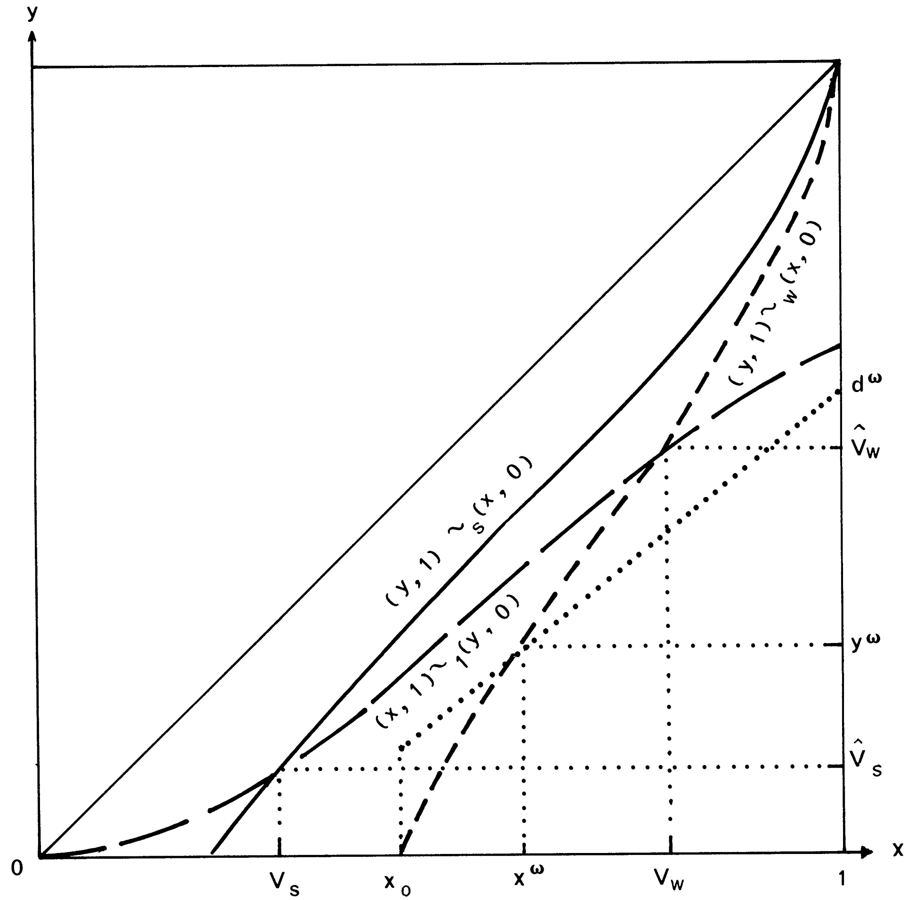

FIGURE 4

Let d_w be the graph of the present value function for 2_w (i.e., $(d_w(y), 0) \sim_w (y, 1)$). For x_0 satisfying $(x_0, 0) \sim_w (0, 1)$, $d^\omega(x_0) = \omega d_1(x_0) + (1-\omega_0) \cdot 0 \geq 0$ and therefore d^ω and d_w must have a nonempty intersection.

The next proposition states that the intersection of d^ω and d_w consists of a single point (x^ω, y^ω). This point plays an important role in the main theorem:

PROPOSITION 7: *There is a unique point (x, y) such that $(x, 0) \sim_w (y, 1)$ and $y = d^\omega(x)$.*

PROOF: Assume that both (x_1, y_1) and (x_2, y_2) satisfy the two equations and $x_1 < x_2$. By definition,

$$(y_i, 0) \sim_1 \omega(x_i, 1) \oplus (1-\omega)(y_i, 2) \quad \text{for} \quad i = 1, 2.$$

By (A-7), $(1-\omega)(u(y_i)-\delta^2 u(y_i))=\omega(\delta u(x_i)-u(y_i))$. Therefore $y_2>y_1$ implies that $\delta u(x_2)-u(y_2)>\delta u(x_1)-u(y_1)$ and $u(x_2)-u(x_1)>u(y_2)-u(y_1)$. The function u is concave and $x_i>y_i$. Therefore, $y_2-y_1>x_2-x_1$, contradicting (A-5) as to $\gtrsim_w$. *Q.E.D.*

7. THE MAIN RESULT

THEOREM: *For a game starting with player* 1*'s offer*: (i): *If* ω_0 *is high enough such that* $y^{\omega_0}>\hat{V}_s$, *then the only bargaining sequential equilibrium outcome is* $\langle (x^{\omega_0},0),(y^{\omega_0},1)\rangle$. (ii): *If* ω_0 *is low enough such that* $y^{\omega_0}<\hat{V}_s$, *then the only bargaining sequential equilibrium outcome is* $\langle (V_s,0),(V_s,0)\rangle$.

In the first case, player 1 offers x^{ω_0}, 2_w accepts this offer, and 2_s rejects it and offers y^{ω_0}, which is accepted. 2_w is indifferent between $(x^{\omega_0},0)$ and $(y^{\omega_0},1)$. In the second case, 1 offers V_s and both types 2_w and 2_s accept it. In the boundary between the two zones more than one bargaining sequential equilibrium outcome is possible.

Let us contrast the equilibrium determined in the Theorem with that derived with complete information: the weak player is better off whereas the strong player suffers a disadvantage in the incomplete information game. It is not clear whether player 1 would benefit from having the information about player 2's type. Delays can occur but only if they result in some information transmission.

EXAMPLE: Assume that 1, 2_w, and 2_s have discounting factors δ, δ_w, and δ_s, respectively. Then

$$V_s=\frac{1-\delta_s}{1-\delta\delta_s},\quad V_w=\frac{1-\delta_w}{1-\delta\delta_w},$$
$$\hat{V}_s=\delta V_s,\quad \text{and}\quad \hat{V}_w=\delta V_w.$$

Here, if

$$\omega_0>\frac{V_s-\delta^2 V_s}{1-\delta_w+\delta V_s(\delta_w-\delta)}=\omega^*$$

we are in case (i) of the theorem and

$$x^{\omega}=\frac{(1-\delta_w)(1-\delta^2(1-\omega))}{1-\delta^2(1-\omega)-\delta\delta_w\omega}.$$

Notice that $1\geqslant\omega^*$ if and only if $V_w\geqslant V_s$.

Where the discounting factors are derived from the continuous discounting formula we get the following limit result when we tend the length of a period to zero: If $\omega_0 > 2r_s/(r_s+r_w)$, the outcome is $\omega_0 r_w/(\omega_0 r_w+(2-\omega_0)r)$, and if $\omega_0 < 2r_s/(r_s+r_w)$, the outcome is $r_s/(r+r_s)$.

The main difficulty here is to prove the uniqueness of the bargaining sequential equilibrium outcome. However, for a better insight of the result, I begin by describing a bargaining sequential equilibrium which induces the outcome $\langle(x^{\omega_0},0),(y^{\omega_0},1)\rangle$ if $y^{\omega_0} > \hat{V}_s$ and induces the outcome $\langle(V_s,0),(V_s,0)\rangle$ if $y^{\omega_0} < \hat{V}_s$.

1's *beliefs*: Player 1 does not change his initial belief unless player 2 has rejected an offer s^{t-1} and offers s^t such that $(s^{t-1},0) \succcurlyeq_w (s^t,1)$; then he changes his belief to $\omega^t(\boldsymbol{s}^t)=0$. (This belief system certainly satisfies (B-1)-(B-2).)

1's *strategy*: Let $\boldsymbol{s}^t$ be the "empty" history or a history that ends with 2's offer s^t. Let $\omega=\omega^t(\boldsymbol{s}^t)$. If $y^\omega \leqslant \hat{V}_s$ (this case includes the possibility that $\omega=0$), then 1 accepts s^t if and only if $s^t \geqslant \hat{V}_s$; otherwise he accepts s^t only if $s^t \geqslant y^\omega$. If 1 rejects s^t (or $\boldsymbol{s}^t$ is the empty history), then he makes the offer x^ω if $y^\omega > \hat{V}_s$ and the offer V_s if $y^\omega \leqslant \hat{V}_s$.

2_w's *strategy*: Let $\boldsymbol{s}^t$ be a history that ends with 1's offer s^t. If $y^\omega \leqslant \hat{V}_s$, then 2_w accepts s^t if and only if $(s^t,0) \succcurlyeq_w (\hat{V}_s,1)$. If $y^\omega > \hat{V}_s$ then 2_w accepts s^t only if $s^t \leqslant x^\omega$. In case 2_w rejects s^t he offers $\hat{V}_s$ if $y^\omega \leqslant \hat{V}_s$ and if $y^\omega > \hat{V}_s$ then he offers the lowest number from among y^ω and the y satisfying $(s^t,0) \succcurlyeq_w (y,1)$ and $y \geqslant \hat{V}_s$.

2_s's *strategy*: Player 2_s always makes the same offer that 2_w does. He accepts only V_s or less.

Whenever $\omega^t(\boldsymbol{s}^t)=0$ or $\omega^t(\boldsymbol{s}^t)=1$, player 1 follows the perfect equilibrium strategy described in [**14**] for games with complete information between 1 and 2_s, or between 1 and 2_w. If $\omega^t=0$, 2_s follows the perfect equilibrium strategy and 2_w chooses a perfect best-response strategy to 1's strategy (and similarly for the case $\omega^t=1$).

8. PROOF OF THE UNIQUENESS OF THE BARGAINING SEQUENTIAL EQUILIBRIUM

Assume that $y^{\omega_0} > \hat{V}_s$; in what follows it is proved that the only bargaining sequential equilibrium outcome is $\langle(x^{\omega_0},0),(y^{\omega_0},1)\rangle$. The proof for the case that $y^{\omega_0} < \hat{V}_s$ is very similar.

Let BSE_1 be a short notation for a bargaining sequential equilibrium in the game where 1 starts the bargaining, and let BSE_2 be a short notation for a bargaining sequential equilibrium in a subgame in which 2 starts the bargaining, if an offer of 1 exists such that 2's reply "N" and continuation according to the BSE_2 is a bargaining sequential equilibrium in the subgame starting after 1's offer. Given a number $0 \leqslant \omega \leqslant 1$, define:

$$A^\omega = \{u \mid (u,0) \sim_w 0_w \text{ where } \langle 0_w, 0_s\rangle \text{ is a } BSE_1 \text{ outcome}\},$$

$$B^\omega = \{v \mid (v,0) \sim_w 0_w \text{ where } \langle 0_w, 0_s\rangle \text{ is a } BSE_2 \text{ outcome}\}.$$

In Section 7 it was proved that $x^{\omega_0} \in A^{\omega_0}$ and $y^{\omega_0} \in B^{\omega_0}$.

LEMMA 1: *$\langle(V_s,0)(V_s,0)\rangle$ is not a BSE_1 outcome and $\langle(\hat{V}_s,0)(\hat{V}_s,0)\rangle$ is not a BSE_2 outcome.*

PROOF: In any of these cases, player 1 would be able to deviate and offer a partition x such that $(x, 0) \sim_w (\hat{V}_s, 1)$. Type 2_w must accept it. Since $y^\omega > \hat{V}_s$, $\omega_0(x, 1) \oplus (1-\omega_0)(\hat{V}_s, 2) >_1 (\hat{V}_s, 0) \sim_1 (V_s, 1)$; thus $\omega_0(x, 0) \oplus (1-\omega_0)(\hat{V}_s, 1) >_1 (V_s, 0)$, and 1 gains by the deviation.

LEMMA 2: *If $v \in B^\omega$ there exists a $u \in A^\omega$ such that $(u, 0) \sim_w (v, 1)$.*

PROOF: Define the following BSE_1: Player 1 offers u, 2_w accepts it, 2_s rejects it and offers v at the next period. Type 2_w rejects any offer higher than u and 2_s rejects any offer higher than V_s. In the case that 1's offer is rejected, the continuation is like the original BSE_2 without a change in 1's belief.

Notice that $\omega_0(u, 1) \oplus (1-\omega_0)(v, 2) >_1 (\hat{V}_s, 0) \sim_1 (V_s, 1)$; thus $\omega_0(u, 1) \oplus (1-\omega_0)(v, 1) >_1 (V_s, 0)$ and 1 cannot gain by offering V_s.

LEMMA 3: *Define $m_1 = \inf A^{\omega_0}$, $m_2 = \inf B^{\omega_0}$; then $(m_1, 0) \sim_w (m_2, 1)$.*

PROOF: By Lemma 2, $(m_1, 0) \succcurlyeq_w (m_2, 1)$. Assume $(m_1, 0) \succ_w (m_2, 1)$. Notice that $t_w = 0$ in every BSE_1 outcome $\langle 0_w, 0_s \rangle$ such that $0_w = (s_w, t_w)$ is $\succcurlyeq_w$ close enough to $(m_1, 0)$; otherwise $(s_w, t_w - 1) \sim_w (v, 0)$ for $v < m_2$, and $(s_w, t_w - 1)$ is a BSE_2 outcome and thus $v \in B^{\omega_0}$. If 1 deviates from the BSE_1 which yields the outcome $\langle 0_w, 0_s \rangle$, and demands x such that $(x, 0) \sim_w (m_2, 1)$, 2_w agrees and 1 gains since $x > m_1$ and, by Lemma 1, the BSE_1 must be of type $(T-1)$. Thus $(m_1, 0) \sim_w (m_2, 1)$.

LEMMA 4: $\omega_0(m_1, 1) \oplus (1-\omega_0)(z(m_1), 2) \sim_1 (m_2, 0)$. (*Recall that* $(x, 0) \sim_w (z(x), 1)$.)

PROOF: The existence of bargaining sequential equilibrium described in Section 5 implies that $m_1 \leqslant x^{\omega_0}$ and $m_2 \leqslant y^{\omega_0}$. Therefore by Lemma 3

$$\omega_0(m_1, 1) \oplus (1-\omega_0)(z(m_1), 2) \succcurlyeq_1 (m_2, 0).$$

Assume that we have a strict inequality. A BSE_2 with an outcome $\langle 0_w, 0_s \rangle$ where 2_w is indifferent between $0_w = (s_w, t_w)$ and an outcome in which he receives a partition whose present value is quite close to m_2, satisfies that $t_w = 0$ (otherwise it contradicts Lemma 3). Thus the BSE_2 has to be of type (T-3). Player 1 can deviate profitably by demanding m_1, accepting it from 2_w, and accepting $z(m_1)$ from 2_s.

LEMMA 5: *$m_1 = x^{\omega_0}$ and $m_2 = y^{\omega_0}$.*

PROOF: Use Lemmas 3 and 4.

LEMMA 6: *Define $M_1 = \sup_{\omega \leqslant \omega_0} A^\omega$ and $M_2 = \sup_{\omega \leqslant \omega_0} B^\omega$; then $(M_1, 0) \preccurlyeq_w (M_2, 1)$.*

PROOF: Similar to the proof of Lemma 2.

LEMMA 7: $(M_1, 0) \sim_w (M_w, 1)$.

PROOF: This is clear if $M_1 = x^{\omega_0}$. Assume $M_1 > x^{\omega_0}$ and $(M_1, 0) <_w (M_2, 1)$. Let $x > x^{\omega_0}$ be close enough to M_1 and $\varepsilon > 0$ small enough such that $x \in A^\omega$ for some $\omega \leq \omega^0$, $z(x) - \varepsilon > M_2$, and $(z(x) - \varepsilon, 0) >_1 \omega_0(M_1, 1) + (1 - \omega_0)(z(M_1), 2)$. There exists a BSE_1 such that 1 offers x and 2_w accepts it (otherwise it contradicts the definition of M_2). Assume 2 deviates and offers $z(x) - \varepsilon$. Player 1 must reject it.

Let $\bar{\omega} \leq \omega_0$ be 1's belief that 2 is 2_w after the offer $z(x) - \varepsilon$. Then

$$(z(x) - \varepsilon, 0) >_1 \bar{\omega}(M_1, 1) \oplus (1 - \bar{\omega})(z(M_1), 2).$$

When rejecting $z(x) - \varepsilon$, 1 must expect an outcome better than $z(x) - \varepsilon$ of a bargaining sequential equilibrium in the subgame after the rejection. The last inequality with Lemma 1 implies that this bargaining sequential equilibrium must be of type (T-3). Therefore there is $y \in B^{\bar{\omega}}$ such that $(y, 2) >_1 (z(x) - \varepsilon, 0)$. Since $z(x) - \varepsilon > M_2$, this contradicts M_2's definition.

LEMMA 8: $\omega_0(M_1, 1) \oplus (1 - \omega_0)(M_2, 2) \sim_1 (M_2, 0)$.

PROOF: By Lemma 7 and $M_1 \geq x^{\omega_0}$,

$$\omega_0(M_1, 1) \oplus (1 - \omega_0)(M_2, 2) \preccurlyeq_1 (M_2, 0).$$

Assume $\omega_0(M_1, 1) + (1 - \omega_0)(M_2, 2) <_1 (M_2, 0)$. Let $y < v < M_2$ satisfying that v is the 2_w's present value of a BSE_2, $\omega_0(M_1, 1) \oplus (1 - \omega_0)(M_2, 2) <_1 (y, 0)$, and $(M_2, 2) <_1 (y, 0)$.

By the definition of BSE_2 there exists an offer of player 1, x, such that 2's reply "N" and continuation according to the BSE_2 is a bargaining sequential equilibrium in the subgame starting with 1's offer. Then if player 2 deviates by refusing x and offering y then player 1 must reject y; otherwise it would be a profitable deviation. This contradicts the fact that by the choice of y, and since 1's belief that 2 is of type 2_w after 2's deviation is lower than ω_0, it is optimal for 1 to accept y.

LEMMA 9: $M_1 = x^{\omega_0}$ *and* $M_2 = y^{\omega_0}$.

PROOF: A conclusion of Lemmas 7 and 8.

9. FINAL REMARKS

The Length of Negotiation

There is a significant difference between the bargaining sequential equilibria in the two cases where $y^{\omega_0} > \hat{V}_s$ and $y^{\omega_0} < \hat{V}_s$. When $y^{\omega_0} < \hat{V}_s$ the bargaining ends at the first period and there is no screening of player 2's type. If $y^{\omega_0} > \hat{V}_s$, then the bargaining in equilibrium might continue into the second period. Further research is needed for clarifying the direction for the generalization of the current result.

The Choice of Conjectures

In [**15**] I study the set of sequential equilibrium outcomes under several other assumptions about the choice of conjectures in the bargaining game with fixed bargaining costs. Specifically, I study the "optimistic conjectures" according to which a deviation of player 2 always convinces player 1 that 2 is of type 2_w. The optimistic conjectures are very often used in the incomplete information bargaining literature since they serve as the best deterring conjectures.

The Game Starts with Player 2's Proposal

In the main theorem the bargaining sequential equilibria are characterized for the case that player 1 opens the game. If player 2 starts the bargaining and if $y^{\omega_0} > \hat{V}_s$, then $\langle (y^{\omega_0}, 0), (y^{\omega_0}, 0)\rangle$ is a BSE outcome and if $y^{\omega_0} < \hat{V}_s$, then $\langle (\hat{V}_s, 0), (\hat{V}_s, 0)\rangle$ is a BSE outcome. This can be verified from the construction of a BSE in the Theorem while noticing that the strategies, after 1 has demanded the whole pie and has been refused, are BSE in a subgame starting with player 2's proposal and initial belief ω_0. However, this is not the only BSE since (B-1)-(B-2) are not effective as restrictions on 1's belief after 2's first offer. By Proposition 4, 2_w and 2_s make the same offer at the first period. We may extend (B-2) so that we require that if 2 was supposed to make the offer y and his offer s^0 was less than y, then $\omega^1(s^0) \leq \omega_0$. With the extension of (B-2) we get that if $y^{\omega_0} < \hat{V}_s$, then $\langle (\hat{V}_s, 0), (\hat{V}_s, 0)\rangle$ is the only BSE outcome. If $y^{\omega_0} > \hat{V}_s$, we get one additional BSE outcome $\langle (y^{\omega_0}, 1), (x^{\omega_0}, 2)\rangle$: player 2 starts with the offer $\hat{V}_s$, which is rejected by player 1 who continues as if he had started the game; if 2 makes an offer $s_0 > \hat{V}_s$, 1 concludes that 2 is 2_w and demands V_w.

Fixed Bargaining Costs

Assume that player 1's utility is $s - ct$ while the utilities of types 2_w and 2_s are $(1-s) - c_w t$ and $(1-s) - c_s t$. The preferences which are represented by these utilities do not satisfy Assumptions (A-5), (A-6) but the same techniques which are used to prove the main theorem are useful to calculate the bargaining sequential equilibrium here:

(i) If

$$\omega_0 > \frac{2c}{c + c_w},$$

the only bargaining sequential equilibrium outcome is $\langle (1, 0), (1 - c_w, 1)\rangle$.

(ii) If

$$\frac{2c}{c+c_w} > \omega_0 > \frac{c+c_s}{c+c_w},$$

the only bargaining sequential equilibrium outcome is $\langle (c_w, 0), (0, 1) \rangle$.

(iii) If

$$\frac{c+c_s}{c+c_w} > \omega_0,$$

the only bargaining sequential equilibrium outcome is $\langle (c_s, 0), (c_s, 0) \rangle$. (The appearance of the intermediate zone (2) is due to the fact that here $(V_s, 1) <_1 (\hat{V}_s, 0)$ and not $(V_s, 1) \sim_1 (\hat{V}_s, 0)$, as in the previous case.)

The Hebrew University of Jerusalem

Manuscript received August, 1983; final revision received October, 1984.

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