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A two period choice problem is presented. An individual has to choose in each period between legal and illegal activities. Two punishment policies are compared: (1) The maximal punishment is imposed even for a first offense. (2) A new offender gets a reduced punishment. It is proved that for a given policy of type 1 there exist a utility function and a policy of type 2 which yields a better deterrent on individual characterized by this utility function.

1. Introduction

Is there an inverse relationship between the incidence of crime and the severity of punishment? A positive answer to this question is obtained in Becker's model [see Becker (1968)]. The individual faces a discrete choice between committing or not committing a specific offense. The monetary gain from the offense, if the offender is not caught, is c. If he commits the offense he may be caught with probability $1-\beta$ and then his net loss is p. The individual's utility function is u. The individual chooses criminal behavior if

$$(1-\beta)u(-p)+\beta u(c)>(0).$$

The left-hand side of this inequality decreases in p. This implies that if p_0 deters the individual, then so will any $p_1>p_0$.

The aim of this paper is to present a certain aspect of criminal behavior which is likely to affect anomalously the deterrent effect of punishment.

In the present model the individual is faced with a two-period choice problem. In each period he must choose between law-abiding and criminal behavior. His decision in the second period may depend on the outcome of his action in the first period. Each of the alternatives involves uncertainty. If he chooses the criminal action he may be caught and punished. Compliance with the law leaves him with uncertainty regarding the level of his income. He may succeed and receive a high income or he may fail and receive only a low one.

The legal system controls the punishment imposed on the convicted criminal. It may impose different punishments on an individual according to his record. A punishment policy is called 'severe' if the maximal sentence is imposed even for a first offense. A punishment policy is called 'lenient' if the first time offender gets a reduced punishment. The punishment policy is common knowledge.

It is proved that for any parameters of the model a utility function can be found according to which an individual, who maximizes expected utility from the sum of his incomes in both periods, is deterred 'more' by a lenient policy than by the severe policy.

This result can be explained intuitively as follows: A lenient policy implies a twofold loss to the convicted individual: the punishment itself, and the loss of the option of committing the illegal action at a later time, when the value of such an action may appear to the individual to be greater. The severe policy increases the direct loss but may decrease the speculative loss by more.

This paper does not pretend to exhaust the subject. Further discussion is needed in order to find the conditions for the existence of the anomaly described above.

2. The model

Denote by W the random variable which takes the value s ($s > 0$) with probability α ($0 < \alpha < 1$) and the value 0 with probability $1 - \alpha$. W is the individual's income if he chooses the legal action.

Denote by C the random variable which takes the value c with probability β and the value $-p$ with probability $1 - \beta$ ($c > s$, $1 > \beta > 0$ and $p > 0$). p is interpreted as the maximal punishment and $1 - \beta$ as the probability of being caught. C is the individual's income if he chooses the illegal action, given that he expects maximal punishment.

Denote by C_1 the random variable which takes the value c with probability β and the value $-p_1$ with probability $1-\beta$ $(0<p_1<p)$. C_1 is interpreted as the income of the individual from the forbidden action if he has never been convicted. C_1 may differ from C only in the punishment the individual expects if he is caught.

The individual is faced with a two-period choice problem which can be reduced to a single period choise problem between eight strategies of the following structure:

'I choose A in the first period. If I succeed, i.e. if $A>0$, I shall choose B_s in the second period, whereas in case of failure (if $A \leqslant 0$), my choice will be B_f', where

$$A \in \{W, C_1\} \quad \text{if} \quad A = W, \quad B_s, B_f \in \{W, C_1\},$$

and if

$$A = C_1,. \ B_s \in \{W, C_1\} \quad \text{and} \quad B_f \in \{W, C\}.$$

Denote such a strategy by $\langle A, B_s, B_f \rangle$.

Assume that the individual has a utility function μ (increasing and continuous) and that the individual maximizes his expected utility from the sum of incomes in the two periods.

Finally assume that the legal policy minimizes the expected number of offenses. Thus, the legal policy problem is

$$\min_{p_1} \text{[expected number of offenses]},$$

subject to the individual maximizes expected utility from the sum of incomes.

3. The anomaly

Proposition. *For all α, β, s, c and p there exists a utility function such that the optimal p_1 is less than p. An individual who is represented by this utility function chooses $\langle W, W, W \rangle$ or $\langle W, W, C_1 \rangle$ where the punishment policy is lenient, and chooses $\langle C, W, C \rangle$ where the punishment policy is severe.*

Proof. The proposition is implied by Lemmas 1 and 2 below.

Let U be the set of all real, increasing and continuous functions (but *not* necessarily concave). Given random variables X, Y, denote

$$X \succsim_u Y \quad \textit{if} \quad Eu(X) \geqslant Eu(y),$$

and

$$X \succ_u Y \quad \textit{if} \quad Eu(X) > Eu(y).$$

If, for all $u \in U$, $X \succsim_u Y$, denote this by $X\ D_1\ Y$ and say that X first degree stochastic dominates Y. Given that $\mathbf{A}$ is a set of random variables, say that $A \in \mathbf{A}$ is u-preferable in $\mathbf{A}$ if $A \succsim_u X$ for all $X \in \mathbf{A}$.

Denote by $A_1, \ldots, A_8$ the random variables representing the sum of incomes from the eight strategies which are available to the individual under the lenient policy and by $\tilde{A}_1, \ldots, \tilde{A}_8$ those available under the severe policy.

The correspondence between the random variables and the strategies is shown in the following table:

i	1	2	3	4	5	6	7	8
A_i	W,W,W	W,W,C_1	W,C_1,W	W,C_1,C_1	C_1,W,W	C_1,W,C	C_1,C_1,W	C_1,C_1,C
$\tilde{A}_i$	W,W,W	W,W,C	W,C,W	W,C,C	C,W,W	C,W,C	C,C,W	C,C,C

Lemma 1. Let $u \in U$ satisfy

$$c + W \succ_u c + C_1, \tag{1}$$

$$s + W \succ_u s + C_1, \tag{2}$$

$$-p_1 + W \succ_u -p_1 + C, \tag{3}$$

$$\tilde{A}_6 \succ_u \tilde{A}_1, \tag{4}$$

$$\tilde{A}_6 \succ_u \tilde{A}_2. \tag{5}$$

Then, A_1 or A_2 are u-preferable in $\{A_1, \ldots, A_8\}$ and $\tilde{A}_6$ is the u-preferable strategy in $\{\tilde{A}_1, \ldots, \tilde{A}_8\}$.

Proof. From (2) it follows that A_1 or A_2 are the u-preferable in $\{A_1,\dots,A_4\}$. From (1) and (3) A_5 is the u-preferable in $\{A_5,\dots,A_8\}$. A_4 and A_5 are equi-distributive and therefore A_1 or A_2 are u-preferable in $\{A_1,\dots,A_8\}$.

$C_1\ D_1\ C$ and therefore (1) implies $c+W \succ_u s+C$, thus $\tilde{A}_5$ or $\tilde{A}_6$ are u-preferable in $\{\tilde{A}_5,\dots,\tilde{A}_8\}$. From (2) $s+W \succ_u s+C$ and therefore $\tilde{A}_1$ or $\tilde{A}_2$ are u-preferable in $\{\tilde{A}_1,\dots,\tilde{A}_4\}$. But $\tilde{A}_4$ and $\tilde{A}_5$ are equi-distributive and from (4) and (5) $\tilde{A}_6$ is the u-preferable in $\{\tilde{A}_1,\dots,\tilde{A}_8\}$.

Lemma 2. *There exist $u \in U$ and $p_1 < p$ satisfy* (1) *to* (5).

Proof. Let p_1 be sufficiently close to p such that $c-p > s-p_1$ and $(c-p)(c-p_1) \geqslant 0$.

It is sufficient to show that there exists $u \in U$ which satisfies conditions (1)–(5) of the lemma. Otherwise, for all $u \in U$ at least one of the following inequalities exists:

$$c + C_1 \gtrsim_u c + W,$$

$$s + C_1 \gtrsim_u s + W,$$

$$-p_1 + C \gtrsim_u -p_1 + W,$$

$$\tilde{A}_1 \gtrsim_u \tilde{A}_6,$$

$$\tilde{A}_2 \gtrsim_u \tilde{A}_6.$$

According to Fishburn (1974, 1975) given random variables $\{x_i\}_{i=1}^k$ and $\{Y_i\}_{i=1}^k$ for any $u \in U$ there exists i $(1 \leqslant i \leqslant k)$ such that $Y_i \gtrsim_u X_i$ if and only there is a convex combination of real numbers $\lambda_1,\dots,\lambda_k$ such that $\Sigma_{i=1}^k \lambda_i Y_i$, $D_1 \Sigma_{i=1}^k \lambda_i X_i$. Therefore there exists $\lambda_1,\dots,\lambda_5$ ($\lambda_i \geqslant 0$ and $\Sigma_{i=1}^5 \lambda_i = 1$) such that $Y = \lambda_1(c+C_1) + \lambda_2(s+C_1) + \lambda_3(-p_1+C) + \lambda_4\tilde{A}_1 + \lambda_5\tilde{A}_2$ first degree stochastic dominates $X = \lambda_1(c+W) + \lambda_2(s+C_1)\lambda_3(-p_1+W) + \lambda_4\tilde{A}_6 + \lambda_5\tilde{A}_6$.

Denote by F_X and F_Y the distributions of X and Y respectively. From the sufficient and necessary condition for first degree stochastic dominance [see for example Rothschild–Stiglitz (1970) and Hanoch–Levy (1969)] $F_X \geqslant F_Y$. An examination of the inequalities at the points of discontinuity of F_X and of F_Y reveals a contradiction [for details, see Rubinstein (1979)].

References

Becker, G., 1968, Crime and punishment, Journal of Political Economy 76, 169–217.

Fishburn, P.C., 1974, Convex stochastic dominance with finite consequence sets, Theory and Decision 5, 119–137.

Fishburn, P.C., 1975, Separation theorems and expected utilities, Journal of Economic Theory 11, 16–34.

Hanoch, G. and H. Levy, 1969, The efficiency analysis of choices involving risk, Review of Economic Studies 36, 335–346.

Rothschild, M. and J.E. Stiglitz, 1970, Increasing risk I, Journal of Economic Theory 2, 225–243.

Rubinstein, A., 1979, On an anomaly of the deterrent effect of punishment, Research Report no. 124 (Department of Economics, The Hebrew University, Jerusalem).