Exam in Microeconomics for Phd.

NYU Economics

Lecturer: Ariel Rubisntein

Date: October 17th

Time: 09:00 - 12:00

Instructions: You are required to answer all three questions. Please use a separate notebook for each question. It is an open-book exam and you can use any written source that you wish. It is not permitted to use electronic devices.

Question 1 (easy): A consumer in a world with $K > 2$ commodities faces a vector of prices $(p_1, \ldots, p_K)$ and is endowed with wealth w.

(a) The consumer chooses the maximal equal quantities he can afford of two cheapest goods. This rule implies the demand correspondence $D(p_1, \ldots, p_K, w)$. Is this correspondence rationalizable?

(b) Is the indirect preference (namely, the one that compares the consumer's preferences at $(p_1, \ldots, p_K, w)$ to those at $(p'_1, \ldots, p'_K, w')$) continuous?

(c) Alternatively, the consumer chooses the maximal equal quantities he can afford of two goods: one cheapest and one most expensive. Is the implied demand correspondence rationalizable?

(d) What can you say about the question in part (b) being asked about the consumer in part (c) ?

Question 2 (medium): Individuals 1 and 2 have preference relations $\succsim^1$ and $\succsim^2$ respectively over the set of lotteries $L(X)$.

We say that *individual* 1 moves in the direction of individual 2 if he adopts a preference relation $\succsim$ which satisfies that for any set of lotteries A and any two lotteries p and q in A such that (i) p maximizes $\succsim^1$ over A and (ii) q maximizes $\succsim$ over A, it holds that $q \succsim^2 p$. In other words, if the change in individual 1's preferences changes his behavior then it will be to the benefit of individual 2.

a) Assume that the individuals are decision makers who maximize the expected utilities u^1 and u^2 respectively. Show that if individual 1 switches to maximizing the expected function $\lambda u^1 + (1-\lambda)u^2$ for some $\lambda \in (0,1)$, then he moves in the direction of individual 2.

b) Assume that each individual i's preferences are represented by the function $U^i(p) = max_{x \in supp(p)}\{u^i(x)\}$, namely he is always optimistic and believes that the best possible outcome in the support of a lottery is realized. Would it always be true that individual 1 moves in the direction of individual 2 if he switches to using the function $\lambda u^1 + (1-\lambda)u^2$ instead of u^1?

Question 3 (more difficult): The number of individuals in the society is fixed snd equal to n. A *data set* is a tuple $D = \langle A, \rhd_1, \ldots, \rhd_n \rangle$ where A is some finite set of social alternatives (it is not fixed) and each $\rhd_i$ is a strict ordering of A (also not fixed). We are interested in a rule that strictly ranks the alternatives for any $\langle A, \rhd_1, \ldots, \rhd_n \rangle$.

A *configuration* is a vector of length n of zeroes and ones. We say that the alternatives a, b satisfy the configuration (c_i) in the data set $\langle A, \rhd_1, \ldots, \rhd_n \rangle$ if $a \rhd_i b$ when $c_i = 1$ and $b \rhd_i a$ when $c_i = 0$. For example, the configuration $(1,0,1)$ is satisfied in $\langle A, \rhd_1, \rhd_2, \rhd_3 \rangle$ if $a, b \in A$, $a \rhd_1 b$, $b \rhd_2 a$ and $a \rhd_3 b$.

A *rule* is a set of configurations.

A rule R and a data set $D = \langle A, \rhd_1, \ldots, \rhd_n \rangle$ induce a binary relation $\succ_{R,D}$ on A defined by: $a \succ_{R,D} b$ if there is a configuration in R which is satisfied by the pair (a, b).

A rule R *is proper* if, for any D, $\succ_{R,D}$ is transitive and asymmetric and for any two alternatives $a, b \in A$, either $a \succ_{R,D} b$ or $b \succ_{R,D} a$.

(a) R_1 is a rule that contains only one configuration $(1, \ldots, 1)$. What is $\succ_{R_1,D}$? Is it proper?

(b) For the case of $n = 4$, consider the rule R_2 which consists of 8 configurations:

$(0,1,1,1)$ and all the configurations $(1, y_2, y_3, y_4)$ except $(1,0,0,0)$.

Show that R_2 is not a proper rule.

(c) Find $2n$ proper rules.

(d) *(more difficult)* Show that there are no other proper rules.

1.

(a) The demand correspondence is rationalizable by the preference represented by the utility function $u(x) = max\{min\{x_i, x_j\} \mid \forall i \neq j\}$.

(b) The above indirect utility function is continuous and thus the indirect preferences are continuous.

(c) The demand correspondence is not rationalizable.

Consider the following example:

$w = 30$, $p^1 = (1,2,5)$, $p^2 = (1,29,5)$.

Then, the demand is: $x(p^1, w) = \{(5,0,5)\}$, $x(p^2, w) = \{(1,1,0)\}$.

If rationalizable by $\succsim$, then $(5,0,5) \succ (1,1,0)$ because $(1,1,0) \in B(p^1, w)$.

But also $(1,1,0) \succ (5,0,5)$ because $(5,0,5) \in B(p^2, w)$, a contradiction.

(d) Since the consumer's demand correspondence is not rationalizable it is not clear what the evaluation of the two budget sets will be based on.

2.

(a) Let $U^i(p) = \sum_{x \in supp(p)} p(x) u^i(x)$, and let $U(p) = \sum_{x \in supp(p)} p(x)[\lambda u^1(x) + (1-\lambda) u^2(x)]$. Notice that we have $U(p) = \lambda U^1(p) + (1-\lambda) U^2(p)$ for all p.

Consider a set of lotteries A. Let p be a maximizer of $\succsim^1$ over A and q be a maximizer of $\succsim$ over A. We have for all r in A both $U^1(p) \geq U^1(r)$ and $U(q) \geq U(r)$, which implies that $U^1(p) \geq U^1(q)$ and $U(q) = \lambda U^1(q) + (1-\lambda) U^2(q) \geq U(p) = \lambda U^1(p) + (1-\lambda) U^2(p)$. Since $U^1(p) \geq U^1(q)$, it follows that $U^2(q) \geq U^2(p)$ and therefore $q \succsim^2 p$. Thus, individual 1 moves in the direction of individual 2.

(b) It is not always true that individual 1 moves in the direction of individual 2 if he switches to using the function $\lambda u^1 + (1-\lambda) u^2$. For example, assume that there are three prizes a, b, c and two lotteries in A where $p = \frac{1}{2} a \oplus \frac{1}{2} b$ and $q = [c]$.

Let $u^1(a) = 5,\ u^1(b) = 0,\ u^1(c) = 3$

$u^2(a) = 0,\ u^2(b) = 4,\ u^2(c) = 3,$

$\lambda = 0.5$, and $u = 0.5 u^1 + 0.5 u^2$.

In that case, we have:

$U^1(p) = 5$, $U^1(q) = 3$,

$U^2(p) = 4$, $U^2(q) = 3$,

$U(p) = 2.5$, $U(q) = 3$.

Then, $p \succ^1 q$ and $q \succ p$ but $p \succ^2 q$.

3.

(a) R_1 is the Pareto domination relation. Therefore, given any data set $D = \langle A, \rhd_1, ..., \rhd_n \rangle$, the induced relation $\succsim_{R_1,D}$ is asymmetric and transitive, but there will be alternatives that $\succsim_{R_1,D}$ does not rank unless all $\rhd_i$ agree.

(b) Consider the data set $D = \langle A, \rhd_1, \rhd_2, \rhd_3, \rhd_4 \rangle$ with $A = \{a, b, c\}$ and:

$a \rhd_1 b \rhd_1 c$

$c \rhd_2 a \rhd_2 b$

$c \rhd_3 b \rhd_3 a$

$b \rhd_4 c \rhd_4 a.$

The pair (a, b) satisfies the configuration $(1,1,0,0)$, while the pair (b, c) satisfies the configuration $(1,0,0,1)$, which means that $a \succ_{R_2,D} b \succ_{R_2,D} c$. However, the pair (c, a) satisfies the configuration $(0,1,1,1)$ and we have $c \succ_{R_2,D} a$, which means that $\succ_{R_2,D}$ is not transitive. Thus, R_2 is not proper.

(c) The dictatorship SWFs and the inverse-dictatorship SWFs constitute $2n$ proper rules. The proper rule that individual i is the dictator consists of all the configurations with $c_i = 1$, while the proper rule that individual i is the inverse-dictator consists of all the configurations with $c_i = 0$.

(d) Assume that R is a proper rule such that for every individual i, there is a configuration $c(i) \in R$ in which $c(i)_i = 1$ and a configuration $d(i) \in R$ in which $d(i)_i = 0$. Assume that the constant $(1, \ldots, 1) \in R$ (the complementary case that $(0, \ldots, 0) \in R$ is treated similarly).

We construct a data set $D = \langle A = \{a_0, a_1, \ldots, a_K\}, \rhd_1, ..., \rhd_n \rangle$ such that $\succ_{R,D}$ violates transitivity.

Assign the agents' preference relations between a_0 and a_1 so as to satisfy $c(1)$. Thus, $a_0 \rhd_1 a_1$.

Expand the relations to a_2 by assigning a_2 to just above or just below a_1 in the ranking of each agent $i \neq 2$ as in $C(2)$ and for agent 2 assign a_2 to below all previously treated alternatives (i.e. a_0 and a_1).

Continue in this way to get a profile of orderings on A such that the relative rankings of a_{k-1} and a_k (for every $1 \leq k \leq K$) are as in $C(k)$ and therefore $a_{k+1} \succ_{R,D} a_k$. However, a_0 is ranked by all agents above a_K and since the configuration $(1, \ldots, 1)$ is in R we conclude that $a_0 \succ_{R,D} a_K$, violating the transitivity of $\succ_{R,D}$.