Exam in Microeconomics for Phd.

NYU Economics

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Date: October 17th

Time: 09:00 - 12:00

Instructions: You are required to answer all three questions. Please use a separate notebook for each question. It is an open-book exam and you can use any written source that you wish. It is not permitted to use electronic devices.

Question 1 (easy): A consumer in a world with $K > 2$ commodities faces a vector of prices $(p_1,\ldots,p_K)$ and is endowed with wealth w.

(a) The consumer chooses the maximal equal quantities he can afford of two cheapest goods. This rule implies the demand correspondence $D(p_1,\ldots,p_K,w)$. Is this correspondence rationalizable?

(b) Is the indirect preference (namely, the one that compares the consumer's preferences at $(p_1,\ldots,p_K,w)$ to those at $(p'_1,\ldots,p'_K,w')$) continuous?

(c) Alternatively, the consumer chooses the maximal equal quantities he can afford of two goods: one cheapest and one most expensive. Is the implied demand correspondence rationalizable?

(d) What can you say about the question in part (b) being asked about the consumer in part (c) ?

Question 2 (medium): Individuals 1 and 2 have preference relations $\succsim^1$ and $\succsim^2$ respectively over the set of lotteries $L(X)$.

We say that *individual* 1 moves in the direction of individual 2 if he adopts a preference relation $\succsim$ which satisfies that for any set of lotteries A and any two lotteries p and q in A such that (i) p maximizes $\succsim^1$ over A and (ii) q maximizes $\succsim$ over A, it holds that $q \succsim^2 p$. In other words, if the change in individual 1's preferences changes his behavior then it will be to the benefit of individual 2.

a) Assume that the individuals are decision makers who maximize the expected utilities u^1 and u^2 respectively. Show that if individual 1 switches to maximizing the expected function $\lambda u^1 + (1-\lambda)u^2$ for some $\lambda \in (0,1)$, then he moves in the direction of individual 2.

b) Assume that each individual i's preferences are represented by the function $U^i(p) = max_{x \in supp(p)}\{u^i(x)\}$, namely he is always optimistic and believes that the best possible outcome in the support of a lottery is realized. Would it always be true that individual 1 moves in the direction of individual 2 if he switches to using the function $\lambda u^1 + (1-\lambda)u^2$ instead of u^1?

Question 3 (more difficult): The number of individuals in the society is fixed snd equal to n. A *data set* is a tuple $D = \langle A, \rhd_1, \ldots, \rhd_n \rangle$ where A is some finite set of social alternatives (it is not fixed) and each $\rhd_i$ is a strict ordering of A (also not fixed). We are interested in a rule that strictly ranks the alternatives for any $\langle A, \rhd_1, \ldots, \rhd_n \rangle$.

A *configuration* is a vector of length n of zeroes and ones. We say that the alternatives a, b satisfy the configuration (c_i) in the data set $\langle A, \rhd_1, \ldots, \rhd_n \rangle$ if $a \rhd_i b$ when $c_i = 1$ and $b \rhd_i a$ when $c_i = 0$. For example, the configuration $(1,0,1)$ is satisfied in $\langle A, \rhd_1, \rhd_2, \rhd_3 \rangle$ if $a, b \in A$, $a \rhd_1 b$, $b \rhd_2 a$ and $a \rhd_3 b$.

A *rule* is a set of configurations.

A rule R and a data set $D = \langle A, \rhd_1, \ldots, \rhd_n \rangle$ induce a binary relation $\succ_{R,D}$ on A defined by: $a \succ_{R,D} b$ if there is a configuration in R which is satisfied by the pair (a, b).

A rule R *is proper* if, for any D, $\succ_{R,D}$ is transitive and asymmetric and for any two alternatives $a, b \in A$, either $a \succ_{R,D} b$ or $b \succ_{R,D} a$.

(a) R_1 is a rule that contains only one configuration $(1, \ldots, 1)$. What is $\succ_{R_1,D}$? Is it proper?

(b) For the case of $n = 4$, consider the rule R_2 which consists of 8 configurations:

$(0,1,1,1)$ and all the configurations $(1, y_2, y_3, y_4)$ except $(1,0,0,0)$.

Show that R_2 is not a proper rule.

(c) Find $2n$ proper rules.

(d) *(more difficult)* Show that there are no other proper rules.