Exam in Microeconomics for Phd.
NYU Economics

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Date: October 20th, 2022
Time: 09:00 - 12:00
Instructions: You are required to answer all three questions. It is an open-book exam and you can use any written source that you wish. Obviously, you are forbidden from communicating with anyone during the exam.

Question 1. Let X be a grand set of alternatives. A decision maker has an arsenal of justifications Λ which he can use to justify his choice. Each element in Λ is a weak preference relation over X and at least one of the members of Λ is a strict preference relation. A choice a from $A \subseteq X$ is Λ-justifiable if $a \in A$ is the *unique* best element in A according to some preference relation in Λ. Define $C_\Lambda(A)$ to be the set of Λ-justifiable alternatives in A.

(i) Is C_Λ always rationalizable? Suggest (and prove) one interesting property that C_Λ satisfies regardless of what Λ is and another that it does not satisfy for some Λ.

A: Property α is satisfied: Let $A \subseteq B$ and let $a \in C_\Lambda(B) \cap A$. This implies that a is the unique best element in B according to preferences in Λ. Since $A \subseteq B$, a continues to be the unique best element in A according to the same preferences. Hence, $a \in C_\Lambda(A)$.

Property β is violated: Let Λ contain: $b \succ^1 a \succ^1 c$ and $c \succ^2 a \succ^2 b$. Then, $a, c \in C_\Delta(\{a,c\})$ and $c \in C_\Delta(\{a,b,c\})$ but $a \notin C_\Delta(\{a,b,c\})$. Thus, the correspondence is not necessarily rationalizable.

(ii) Given a choice correspondence C, is there necessarily a set of justifications Λ such that $C = C_\Lambda$?

A: No! Because property α is valid for every Λ.

Now consider a choice function C built on potential justifications ordered by priority $\geq_1, \ldots, \geq_K$. Assume that the lowest priority justification, $\geq_K$, is a strict ordering. The function C selects from A the alternative which is justified by the highest priority justification.

(iii) Is this choice function necessarily rationalizable?

A: Let Λ contain $a \sim^1 b \succ^1 c$ and $c \succ^2 a \succ^2 b$. The choice from $\{a,b,c\}$ is c but from $\{a,c\}$ it is a. [This is also an example for β above.]

(iv) Suggest (and prove) a non-trivial property that this choice function satisfies regardless of what Λ is.

A: If x and y are not in A, $C(A \cup \{x\}) = x$ and $C(A \cup \{x,y\}) \neq x$, then $C(A \cup \{y\}) \in \{x,y\}$.

Question 2. In this question, you are asked to rewrite a "consumer chapter" for a world in which the consumer faces a set X of K indivisible objects and chooses a subset of X. Given a budget w and a price vector $p = (p_k)_{k \in K}$, the consumer can purchase any subset with a total cost of not more than w. Assume that the consumer has a strict preference $\succsim$ on the set Y of subsets of X with the monotonicity property that "adding an item cannot hurt".

a. Formulate the consumer problem.

Each collection of objects Y can be identified as the set $\{0,1\}^K$ where for each $y \in Y$ the term $y^k = 1$ means that the collection y includes k. The consumer is seeking the $\succsim$-maximal collection within the set $B(p,w) = \{y \mid p \cdot y \leq w\}$.

b. Prove that the demand for good k is non-increasing in p_k.

Let p and q be two identical price vectors with the exception that $q_k > p_k$. Let y be demand at price p and wealth w. Obviously, $B(q,w) \subseteq B(p,w)$. If $y_k = 0$, then $y \in B(q,w)$ and remains optimal in $B(q,w)$. If $y_k = 1$, then the demand under q cannot increase.

c. Is it true that all goods are always normal (that is, their demand is non-decreasing in w)?

No. For example, assume that $K = \{a,b\}$, $p_a = 1$, $p_a = 3$ and the preferences are $\{a,b\} \succ \{b\} \succ \{a\} \succ \emptyset$. For $w = 2$, the consumer purchases a and with $w = 4$ he purchases b. Thus, the demand for a is not increasing in w.

d. How would you derive demand from the indirect preference defined over the space of all (p,w)?

Compare (p,w) to (q,w) where q is identical to p with the exception that if $q_k = w + 1$ then $y_k(p,w) = 0$. Since $B(q,w) \subseteq B(p,w)$ it follows that (p,w) is at least as good as (q,w). If the consumer is indifferent between the two then $y_k(p,w) = 0$. If the consumer prefers (p,w) over (q,w), he cannot purchase $x(p,w)$ in (q,w) and this is possible only if $x_k(p,w) = 1$.

e. Assume now that the price vector is such that the prices of any two subsets of goods are distinct. Prove the following duality proposition: y^* is an optimal subset given p and w which is equal to the cost of y^* if and only if y^* is the cheapest set given p which is at least as good as y^*. Explain why the proposition may be incorrect without the assumption (*) that the costs of all subsets are distinct.

If y^* is the best subset in $B(p, w)$ and $p \cdot z < p \cdot y^*$ then $z \in B(p, w)$ and thus $z \prec y^*$. If y^* is the cheapest object given p in $\{y \mid y \succsim y^*\}$ and $z \neq y^*$ is in $B(p, p \cdot y^*)$, then $pz < py^*$ (by (*)) and $z \prec y^*$.

The claim is not true without (*): If $K = \{a, b\}$, $p_a = p_b = 1$ and $\{a, b\} \succ \{b\} \succ \{a\} \succ \emptyset$, then the minimal wealth needed to purchase a set which is at least as good as $\{a\}$ is 1 but with wealth 1 the consumer can buy the set $\{b\}$ which is better.

Question 3. A society has $n \geq 3$ individuals. Let X be a set of social alternatives. For any profile of strict preference relations on X, we wish to attach a "representative" preference relation that is one of the profile's preferences. We use a distance function d over the set of preference relations and define $F(\succsim^1, \ldots, \succsim^n)$ to be the set of preference relations in the profile that minimize the average distance from all preferences in the profile.

(a) Can this correspondence be thought of as a choice correspondence (from sets of preference relations)? (yes/no and a one sentence explanation.)

A: No! Multiple entries of the same preferences affect the choice in this correspondence, whereas in a choice correspondence the number of times an element appears in the description of a set doesn't affect the choice.

(b) Characterize the correspondence F for the case in which d assigns the value 1 to any two distinct preference relations and 0 otherwise.

A: F assigns to a profile the set of the most frequent preference relations.

(c) Characterize F for the case in which $X = [0, 1]$, each preference relation has a single peak and the distance between two preference relations is defined as the distance between their peaks.

A: F always picks M, which is the median point between the peaks. By definition, the number of peaks that are $> M$ ($< M$) is less than the number of peaks that are $\leq M$ ($> M$).

Consider $x > M$ (a similar argument can be stated for $x < M$). Note that $\sum_{peak^i \leq M} d(x, peak^i) - \sum_{peak^i \leq M} d(M, peak^i) > n/2(x - M)$ and $\sum_{peak^i > M} d(M, peak^i) - \sum_{peak^i > M} d(x, peak^i) < n/2(x - M)$. Thus, the overall sum of distances is larger for x than it is for M.

(d) Assume that X is finite, all preferences are strict and the (Kemeny) measure distance between any two preferences is the number of pairs for which the two preferences differ. A Social Welfare Function is derived by breaking ties according to some pre-specified order over the orderings. Does this SWF satisfy: (i) the Pareto property; (ii) the IIA property?

A: Obviously this SWF satisfies Par. If it satisfied IIA, then (by Arrow's theorem) it would be a dictatorship. But F is not dictatorship: it assigns $\succsim$ to every profile for which all agents have $\succsim$ except a unique agent who holds $-\succsim$, and thus is not a dictatorship.