Exam in Microeconomics for Phd.
NYU Economics

Lecturer: Ariel Rubisntein
Date: October 21st, 2021
Time: 09:00 - 12:00
Instructions: You are asked to answer the three questions. It is an exam with "open-books" and you can use any written resources. Obviously you are forbidden from communicating with anybody during the exam.

Question 1. Let A be a set of at least three objects. A distance function d assigns a number in $[0, 1]$ to every pair of objects such that for every $x, y, z \in A$

$d(x, x) = 0$; $d(x, y) = d(y, x)$; and $d(x, y) + d(y, z) \geq d(x, z)$.

Let $N = \{1, .., n\}$ be a set of individuals. Each individual i holds a distance function on A.

An aggregator F assigns a distance function to every profile of distance functions $(d_1, .., d_n)$.

An aggregator is *simple* if there exists a function f which assigns a number in $[0, 1]$ to every vector of distances $(\alpha_1, ..., \alpha_n)$ such that:

(i) f generates F in the sense that $F(d_1, .., d_n)(x, y) = f(d_1(x, y), \ldots, d_n(x, y))$.

(ii) *Unanimity:* $f(c, \ldots, c) = c$ for all $c \in [0, 1]$

(iii) *Anonymity:* $f(\alpha_1, \ldots, \alpha_n) = f(\beta_1, \ldots, \beta_n)$ if $(\beta_1, \ldots, \beta_n)$ is a permutation of $(\alpha_1, \ldots, \alpha_n)$.

(a) Assume that the distance between any two objects must be either 0 or 1. Find a simple aggregator.

(b) Assume that the distance between two objects can be any number in $[0, 1]$. Find another example of a simple aggregator.

(c) Show that there is only one correct answer to (a).

Question 2. Professors of economics evaluate job candidates. The set of candidates is large but eventually the choice will be made from a subset containing an odd number of candidates. Professors care only about the candidate's attitude towards behavioral economics, which is measured by a non-zero real number (for example, $+3$ is more positive than $+1$ and -50 is negative). Let $X = \mathit{Reals} \setminus \{0\}$. Assume that no two candidates have the same attitude. Thus, each professor can be described as a choice function that selects one number from each odd cardinality set of numbers.

Both professors wish to select a candidate who best reflects the mood of the profession. Both believe that the attitude to behavioral economics is distributed according to a Normal distribution: either $N(+2,1)$ or $N(-2,1)$.

When facing a set of candidates Y:

Professor A finds the distribution that best explains Y in the sense of maximum likelihood (that is, $N(+2,1)$ if $\sum_{x \in Y} x > 0$, and $N(-2,1)$ if $\sum_{x \in Y} x < 0$) and then chooses the candidate in Y who is the closest to the peak of that distribution.

Professor B divides Y into two groups: positive candidates (closer to $+2$) and negative candidates (closer to -2). He chooses the candidate in Y with a positive attitude who is closest to $+2$ if the majority of candidates are positive, and the candidate with a negative attitude who is closest to -2 if the majority are negative.

(0) Formalize the two professors as choice functions c_A and c_B.

(1) Is either professor rationalizable as a maximizer of a preference relation on X (the set of possible attitudes toward b.e.)? Prove your answer.

(2) Show that the following property of choice functions is satisfied by the two professors: For any $X \supseteq Y_1 \supset Y_2 \supset Y_3$, if $a = c(Y_1) \neq c(Y_2) = b$ and $a, b \in Y_3$, then $c(Y_3) \in \{a, b\}$.

(3) Show that the following property distinguishes between the two professors: If $c(Y) = c(Z) = a$ and $Y \cap Z = \{a\}$, then $c(Y \cup Z) = a$ (that is, the property is satisfied by one professor but not the other).

Question 3. A *test* is a finite vector of 0's and 1's. Let X be the set of tests. The interpretation of the test $(1,0,1,1)$ (for example) is that a student will be asked to answer one of four questions; he fails the test if asked the second and passes if he asked one of the other three. Define $n(s)$ to be the number of possible questions in test s (i.e. the length of the vector s).

A compound test $s \oplus t$ (where $s, t \in X$) is a test in which the student will be given one of the two tests s or t. The student identifies the compound test as a test (vector) of length $n(s_1) + n(s_2)$ where the vector s is first and the second is t. Thus, for example, if $s = (1,1,1)$ and $t = (1,0)$ then $s \oplus t = (1,1,1,1,0)$.

Let $\succsim$ be a preference relation over X satisfying the following properties:

Symmetry: if s is a permutation of t then $s \sim t$.

Monotonicity: any sequence of ones is better than any sequence of zeroes.

Independence (I): if $s \succsim t$ if and only if $s \oplus r \succsim t \oplus r$ for every tests r, s, t.

(a) Interpret I. Explain why $\succsim$ cannot be the preference relation with the utility representation $u(s) =$"the proportion of ones in s".

(b) Given one example of the many preference relations that satisfy the three properties.

(c) Show that it is impossible that both $(0,0) \sim (0)$ and $(1,1) \sim (1)$.

(d) Show that there is only one preference relation satisfying the three assumptions and $(1,1) \sim (1)$.