Solution to Exam in Microeconomics Theory
NYU, Fall 2020,
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Question 1. Suppose $x = f(E)$ is the $\succsim$-maximal path consistent with E. Since n is on the path x then $E \cup \{n\}$ is an evidence set consistent with x. Every path that is consistent with $E \cup \{n\}$ is also consistent with E. Thus as x was the $\succsim$-maximal path consistent with E it is also $\succsim$-maximal path consistent with $E \cup \{n\}$, and $f(E \cup \{n\}) = x$.

b) Define $x \succ y$ if there is some evidence E compatible with both x and y such that $x = f(E)$.

The relation is asymmetric: Assume that there are evidence sets E and E' such that the two nodes x, y are compatible with E and E' and $f(E) = x$ and $f(E') = y$. Then by $f(E) = x$ and the stickiness property $f(E \cup E') = x$. By $f(E') = y$ and the stickiness property $f(E \cup E') = y$. A contradiction.

The relation is transitive: Assume that $x \succ y$ and $y \succ z$ and let E and E' be evidence sets which are consistent with x,y and y, z accordingly such that $f(E) = x$ and $f(E') = y$. All nodes in E are on y not after a_{xy} - the node where x and y split. All nodes of E' are on y not later than a_{yz} the node where y and z split. If a_{yz} would be weakly before a_{xy} than x would be consistent with E' and thus $y \succ x$ contradicting the asymmetry. Thus, a_{yz} is strictly after a_{xy} and thus z is consistent with E and $x \succ z$.

Now that we have seen that the relation $\succ$ does not have cycles we conclude it can be completed to an ordering $\succ^*$ and by being an extension of $\succ$ the path $f(E)$ is always the $\succ^*$ -maximal path consistent with E.

c) Suppose the CB initially (based on observing O) believes $f(\{O\}) = (O, a, b)$. But conditional on observing $E = \{O, a\}$ he changes his mind to $f(\{O, a\}) = (O, a, c)$. This CB doesn't obey stickiness, hence is not order-based.

Question 2.

a) Given a preference, define

$$d(a,b) = \begin{cases} 1 & a \succ b \\ 0 & b \succeq a \end{cases}$$

In this case $u(a;A) := \sum_{x\in A} d(a,x) = |\{x \in A; a \succ x\}|$. Since the preference is transitive, for $a, b \in A$ we have $a \succ b$ iff $\{x \in A; a \succ x\} \supsetneq \{x \in A; b \succ x\}$ iff $u(a;A) > u(b;A)$.

b) Let $X = \{a, b, c\}$ and let

$$d(a,b) = 1 \qquad d(b,c) = 1 \qquad d(c,a) = 2$$

The generated choice function is $C(X) = c$ but $C(\{b,c\}) = b$, contradicting condition α necessary for rationalization.

c) Let $X = \{a, b, c\}$ and consider the choice function

$$C(X) = a \qquad C(\{a,b\}) = b \qquad C(\{b,c\}) = b \qquad C(\{c,a\}) = c$$

If this were described by such a process then $d(a,b) = d(a,c) = 0$ while $d(b,c) > 0$ thus $u(a;X) = 0 < u(b;X)$ implying $C(X) \neq a$,

d) A general property is if $C(\{x,a\}) = x$ for all $x \in A - \{a\}$ then $C(A) \neq a$.

Question 3.

a) The middleman's problem is

$$\max\{(q-p)\cdot x\ ;\ p\cdot x \leq M\}$$

which as a linear problem which has a corner solution. If $p_k \geq q_k$ for all k then the solution is 0. If there is a commodity k for which $q_k > p_k$ then let k^* be a maximizer of q_k/p_k then he trades only with k^*. Formally,

$$x_k(q,p,M) = \begin{cases} \frac{M}{p_{k^*}} & q_{k^*} > p_{k^*} \text{ and } \frac{q_{k*}}{p_{k*}} \geq \frac{q_k}{p_k} \text{ for all } k \\ 0 & \text{otherwise} \end{cases}$$

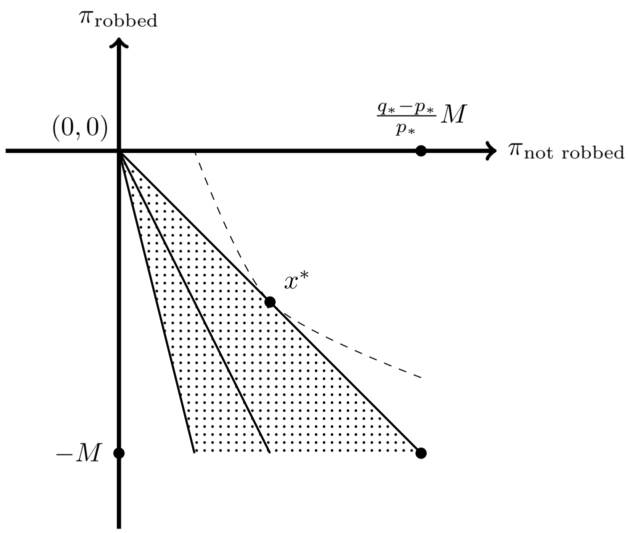
b) The middleman spends all money on the item with highest return (if positive) making a natural indirect utility

$$v(p,q,M) = \begin{cases} \frac{q_k}{p_k} M - M & q_{k^*} > p_{k^*} \text{ and } \frac{q_{k*}}{p_{k*}} \geq \frac{q_k}{p_k} \text{ for all } k \\ 0 & \text{otherwise} \end{cases}$$

The derivative of v with respect to q_k gives you either M/p_k for the good which is traded or 0 for any good which is not traded.

c) In the graph below, we show the profit π the agent makes in the two states with $K=3$. The origin is if he does not purchase any goods and keeps his cash, while diagonal lines are lotteries corresponding to only spending money on one of the goods and saving the rest; they have slopes $(\frac{q_k - p_k}{p_k}, -1)$. The dotted triangle is his choice set when he combines trades with multiple types of goods. Clearly the triangle will be dominated by lotteries with only trading the most profitable good.

If the agent is sufficiently risk averse, he may opt to retain some wealth in cash as insurance against theft. Such a possibility is demonstrated in the picture.


π_{robbed}
$(0,0)$
$\frac{q_*-p_*}{p_*}M$
$\pi_{\text{not robbed}}$
x^*
$-M$