Exam in Microeconomics for Phd.
NYU Economics

Lecturer: Ariel Rubisntein
Date: October 20th, 2020
Time: 09:00 - 12:00 (New York time)
Instructions: You are asked to answer the three questions. It is an exam with "open-books" and you can use any written resources. Obviously you are forbidden from communicating with anybody during the exam.

Question 1 An individual has in mind a directed tree $\langle V, \rightarrow \rangle$with the root O. Each node in the tree stands for an event. The fact that $x \rightarrow y$ means that the event x can be followed by the event y. Denote by X the set of paths of the tree (a path starts from the root of the tree and reaches a terminal node). One and only one of the paths is the true one. The individual receives evidence in the form of a partial set of nodes that have occurred. The evidence is not contradictory in the sense that there is at least one path that includes all the nodes. He selects a conjecture about the true path which is consistent with the evidence.

Formally, define a conjecture-builder (CB) as a function f that attaches to every set of nodes E, which contains O and is non-contradictory, a path $f(E)$ that includes all nodes in E.

a. We say that the CB is "order-based" if there is a preference relation $\succsim$ over X such that $f(E)$ is the $\succsim$ -maximal path that is consistent with E. Show that an order-based CB satisfies the following stickiness property: If the node n is on the path $f(E)$, then $f(E \cup \{n\}) = f(E)$.

b. Show that any CB who satisfies the stickiness property is order-based.

c. Consider the tree with $V = \{O, a, b, c\}$ and $O \rightarrow a$, $a \rightarrow b$, and $a \rightarrow c$. Which CB cannot be presented as order-based?

Problem 2. A decision maker uses the following procedure to choose from any subset of alternatives in the finite set X: For any a and b, he has in mind a number $d(a,b) \geq 0$ such that $d(a,a) = 0$ and if $d(a,b) > 0$ then $d(b,a) = 0$. The number $d(a,b)$ in the case that $d(a,b) > 0$ is a measure by which a defeats b. From a set A, he chooses the maximizer of $\sum_{x \in A} d(a,x)$ (assume that there is always a unique maximizer).

a. Show that if the decision maker is "rational',' then he can be presented as if he activates such a procedure.

b. Give an example of a choice function that is obtained by the procedure but is not rationalizable.

c. Construct an example of a choice function that is not explicable using this procedure.

d. Invent a property of a choice function that is necessary for being explicable by the above procedure.

Problem 3. Middleman A middleman is operating in a world with K divisible goods. He faces two sets of prices p and q where p_k is the price he has to pay for one unit of good k and and q_k is the price he receives when he sells one unit of good k. He starts the day in village A where he can buy goods at prices p, and travels to B where he faces the price vector q. He begins the day with a limited amount of money M and he aims to end the day with as much money as possible.

(a) Formulate the middleman problem and derive his demand function (as a function of p, q, M).

(b) Derive an indirect utility function for the middleman. How would you derive the middleman's choice from the indirect utility function?

(c) Assume that the middleman faces uncertainty: with probability α he expects to be robbed on the way from A and B and all his goods will be stolen (not his money – which he leaves at A). Assume he behaves consistently with expected utility theory and is risk averse. Draw a diagram in which the horizontal axis is the money he will have at the end of the day if he is not robbed and the vertical axis is the money he will have at the end of the day if he is robbed. Explain (diagramatically) why it is possible that he will actively trade only one of the goods and will not use the entire amount of cash.