Ariel Rubinstein: **Micro-Theory**: **NYU**

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Exam

Q1: Consider an economic agent with preferences $\succsim^1$ on the set of the bundles in a K-commodity world. The agent holds a bundle x^* and can consume any part of it; however, he feels obliged to give to his friend (who holds the preference relation $\succsim^2$) a bundle which will be at least as good for his friend as a fixed bundle y^*. Assume that $x^*_k > y^*_k$ for all k. Both preference relations satisfy strong monotonicity, continuity and strict convexity.

(1) State the agent's problem and explain why a solution exists and is unique.

(2) Denote the bundle the agent consumes given x^* as $z(x^*)$. The agent's indirect preferences on the space of initial bundles can be defined by $a^* \succsim^* b^*$ if $z(a^*) \succsim^1 z(b^*)$. Show that the indirect preferences are strictly convex and continuous.

(3) Show that if $\succsim^1$ is differentiable then so is $\succsim^*$.

Q2: A decision maker who compares vectors (x_1,x_2) and (y_1,y_2) in R^2_+ is implementing the following procedure, denoted by $P(v_1,v_2)$, where for $i = 1,2$, v_i is a strictly increasing continuous function from the nonnegative numbers to the real numbers satisfying $v_i(0) = 0$:

(1) if one of the vectors dominates the other he evaluates it being superior.

(2) if $x_1 > y_1$ and $y_2 > x_2$, he carries out a "cancellation" operation and then makes the evaluation by comparing $(x_1 - y_1, 0)$ to $(0, y_2 - x_2)$, which is accoplished by comparing $v_1(x_1 - y_1)$ with $v_2(y_2 - x_2)$ (similarly, if $x_1 < y_1$ and $x_2 > y_2$, he bases his preference on the comparison of $v_2(x_2 - y_2)$ to $v_1(y_1 - x_1)$).

(a) Verify that if $v^*_i(t) = t$ (for both i), then the procedure $P(v^*_1, v^*_2)$ induces a preference relation on R^2_+.

(b) This procedure does not necessarily induce a preference relation. However, if the procedure $P(v_1,v_2)$ induces a preference relation, then that preference relation is represented by.... (complete the senetence and prove it).

Question 3

Discuss the attitude of an agent towards lotteries over a set of consequences $Z = \{a, b, c\}$ satisfying that he ranks a first and c last.

Consider any preference relation (on $L(Z)$) satisfying independence and continuity. Obviously, each preference relation can be described by a single number $v \in (0,1)$ by attaching the numbers $1, v, 0$ to the three alternatives. Denote this preference relation by $\succsim_v$.

For a set $V \subseteq (0,1)$, define a choice correspondence $C_V(A)$ as the set of all $p \in A$ satisfying that there is no $q \in A$ such that $q \succ_v p$ for all $v \in V$.

Define the binary relation pD^*q if $p(a) \geq q(a)$ and $p(a) + p(b) \geq q(a) + q(b)$ with at least one strict inequality. Consider the choice correspondence C defined by $p \in C(A)$ if there is no $q \in A$ such that qD^*p. Show that $C = C_V$ for some set V.