NYU: **Micro Economics for Phd**: **Ariel Rubinstein** :**Exam** - **October 2015**

Q1. Consider a decision maker on the space $X = [0,1]$ where $t \in X$ is interpreted as the portion of the day he contributes to society.

(1) Assume that the decision maker has a strictly convex and continuous preference relation over X. Show that he has a "single peak" preference relation, namely there exists x^* such that for every $x^* \leq y < z$ or $z < y \leq x^*$ he strictly prefers y to z. Find a strictly convex preference relation on this space which is not continuous.

(2) Assume that the domain of the decision maker's choice function contains all sets of the form $B(w,\rightarrow) = \{x \in X \mid x \geq w\}$ as well as all sets of the form $B(w,\leftarrow) = \{x \in X \mid x \leq w\}$, where $w \in [0,1]$. Interpret those sets. Show that the decision maker's choice function induced from a strictly convex and continuous preference relation is always well-defined and continuous in w.

Q2. Consider two types of decision makers:

Type A has in mind several criteria $(\succ_i)_{i\in I}$ where each $\succ_i$ is an ordering of the elements in a finite set X. Whenever the agent has to choose from a set $A \subseteq X$ he is satisfied with any element a such that for any other $b \in A$ there is some i (i probably depends on b) for which $a \succ_i b$.

Thus, for example if he has one criterion in mind then the induced choice correspondence selects the unique maximal element from each set; if he has two criteria in mind, where one is the negation of the other, then the induced choice correspondence is $C(A) \equiv A$.

(1) Show that if $a \in C(A) \cap C(B)$, then $a \in C(A \cup B)$.

(2) Suggest another interesting property that the choice correspondence induced by the above procedure always satisfies.

Type B has in mind a transitive asymmetric relation $\succ$ with the interpretation that if $a \succ b$ then he will not choose b if a is available. He is described by the choice correspondence $C(A) = \{x \in A |$ there is no $y \in A$ such that $y \succ x\}$.

(3) Show that any type A agent can be described as a type B agent.

(4) Show that every type B agent can be described as a type A agent.

Q3. Define an "amount of money" to be any positive integer. Define a "wallet" to be a collection of amounts of money. Denote the wallet which contains the K amounts of money $x_1,..,x_K$ by $[x_1,..,x_K]$. Thus, for example, the wallet [3,3,4] with a total of 10 equal to the wallet [4,3,3] and is different from the wallet [3,4] which has a total of 7. Let X be the set of all wallets. Following are two properties of preference relations over X :

Monotonicity:

(i) Adding an ammount og money to a wallet or increasing one of the existing amounts of money is weakly improving.

(ii) Increasing all the amounts in a wallet is strictly improving.

Split-aversion:

Combining two amounts of money is (at least weakly) improving (thus [7,3] is at least as good as [4,3,3]).

(1) Let v be a strictly increasing function defined on the natural numbers satisfying (i) $v(0) = 0$ and (ii) superadditivity ($v(x+y) \geq v(x) + v(y)$ for all x,y). Show that the function $u([x_1,\ldots,x_K]) = \Sigma_{k=1,,..,K} v(x_k)$ is a utility function which represents a preference relation on X that satisifies monotonicity and split-aversion.

(2) Give an example of a preference relation satisfying monotinicity but not split-aversion and one of a preference relation satsifying split- aversion but not monotonicity.

(3) Define the notion "one preference relation is more split averse than another".

(4) Find a preference relation (satisfying monotonicity and split-aversion) which is less split averse than any other split averse and monotonic preference relation.

(5) Show that the relation represented by the function $u([x_1,\ldots,x_K]) = \max\{x_1,..,x_K\}$ is more split averse than any preference relation of the type described in part (1).