NYU: **Micro Economics for Phd**: **Ariel Rubinstein** :**Exam - October 2014**

Question 1:

Consider the following family of preference relations defined over $L(Z)$ (the set of all lotteries with prizes in some finite set Z):

The DM has in mind a function which assigns to each prize $z \in Z$ a value $v(z)$.

He partitions Z into two sets G and B such that if $g \in G$ and $b \in B$ then $v(g) > v(b)$.

He evaluates any lottery p by

$p(Supp(p) \cap G) \max_{z \in Supp(p) \cap G} v(z) + p(Supp(p) \cap B) \min_{z \in Supp(p) \cap B} v(z)$.

These evaluations form his preferences over $L(Z)$ (where $p(A) = \sum_{z \in A} p(z)$).

0. Explain the procedure in words.

a. Show that such a preference relation satisfies neither the Independence axiom nor the Continuity axiom.

b. Show that a weaker independence property holds: If $Supp(p) = Supp(q)$ then for every $1 > \alpha > 0$ and every r,

$p \succsim q$ iff $\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$.

c. Describe in words and then formally define a "monotonicity property" that holds.

d. (Bonus) Suggest an interesting property that this kind of preferences satisfies.

Question 2:

Let X be a finite set of objects. A betweenness relation B is a 3-place relation on X (presented as a subset of X^3) such that if $(a,b,c) \in B$ then a,b,c are distinct. The interpretation of $(a,b,c) \in B$ is that "b is between a and c".

The following are three possible properties of a betweenness relationship:

A1: If $(a,b,c) \in B$ then $(c,b,a) \in B$.

A2: If $(a,b,c) \in B$ and $(b,d,c) \in B$ then $(a,d,c) \in B$ and $(a,b,d) \in B$. If $(a,b,c) \in B$ and $(b,c,d) \in B$ then $(a,c,d) \in B$ and $(a,b,d) \in B$.

A3: For every a,b,c exactly one of the triples (a,b,c), (b,c,a), (c,a,b) belongs to B.

a. Give three examples to show the "independence" of A1, A2 and A3.

b. Show that if a 3-place relation B satisfies A1, A2 and A3, then there is a function $\alpha : X \to R$ (the real numbers) such that $(x,y,z) \in B$ if and only if the number $\alpha(y)$ is between the numbers $\alpha(x)$ and $\alpha(z)$.

Question 3:
A DM needs to decide how to allocate a budget between two activities: 1 and 2. A combination of activities is a pair (a_1, a_2) where a_i is the level of activity i. The DM's problem is to choose a combination of activities given a budget w and a vector of prices for the activities (p_1, p_2).
Two consultants, A and B, are involved in the DM's process. Each consultant submits to the DM a recommendation which is the outcome of maximizing a "classical" and differentiable preference relation defined over the set of all activity combinations. Assume that whatever the "budget set" is, consultant A always recommends a higher level of activity 1 than B does. Formally, assume that at each combination of activities (a_1, a_2) the "marginal rate of substitution" (the ratio of local values) of A is strictly larger than that of B.

The DM collects the two recommendations and then:

If both recommend that the level of a certain activity i should be higher than that of the other activity, then the DM follows the more "moderate recommendation", namely the one which is closer to the main diagonal.

If consultant A recommends a higher level of activity 1 and B recommends a higher level of activity 2, then the DM spends his entire budget such that he consumes equal levels of the two activities (i.e., a combination on the main diagonal).

a. Assume that A aims to maximize $2a_1 + a_2$ (and in the case of indifference recommends only activity 1) and B seeks to maximize $a_1 + 2a_2$ (and in the case of indifference recommends only activity 2). Is the DM's behavior rationalizable in the sense that there exists a convex and monotonic preference relation that rationalizes the DM's behavior?
b. (bonus) Extend your answer to any two consultants that satisfy the question's assumptions.