Microeconomics I Midterm (Fall 2013)
Ariel Rubinstein
October 17, 2013
First answer the three questions without the Take Home parts. If you have time, answer one of the Take Home parts in either question 1 or 2 (not 3).

Question 1:

Consider the following procedure which yields a choice function C over subsets of a finite set X:

The decision maker has in mind a set $\{\succ_i\}_{i=1,..n}$ of orderings over X and a set of weights $\{\alpha_i\}_{i=1,..n}$. Facing a choice set $A \subseteq X$, the decision maker counts for each alternative $x \in A$ a score: the sum of the weights of those orderings which rank x first from among the members of A and then chooses the alternative which gets the highest score.

To Warm Up:

a) Explain why a rational choice function is consistent with this procedure.

b) Give an example to show that the procedure can produce a choice function which is not rationalizeable.

And now to the main question:

c) Show that for $|X| = 3$ all choice functions are consistent with the procedure.

d) Explain why it is not generally true that a choice function C which is derived from this procedure satisfies the condition that if $x = C(A) = C(B)$ then $x = C(A \cup B)$.

e) (Take Home) Can you find any non-trivial property which is satisfied by choice functions which are derived by this procedure and not by the other? Is there any choice function which cannot be explained by this procedure?

Question 2:

Imagine a consumer who operates in two stages when he faces a budget set $B(p,w)$ in a world with the commodities 1,...,K split into two exclusive non-empty groups A and B :

Stage 1: He allocates w to the two groups by maximizing a function v on the set of pairs (w_A, w_B).

Stage 2: He chooses an A-bundle maximizing a function u_A defined over the A-bundles given w_A, and separately he chooses a B-bundle maximizing a function u_B defined over the B-bundles given w_B.

a) Show that if the consumer is interested to choose at the end a bundle (over the K commodities) which maximizes the (ridiculous) utility function $\Pi_{k=1,..,K} x_k^{\alpha_k}$ (where $\alpha_k > 0\ \forall k$ and $\sum_{k=1}^{K} \alpha_k = 1$) then he can attain his goal by following the procedure above with some functions (v, u_A, u_B).

b) Show that the claim in (a) is not true in general. For example, you might (but don't have to) look at the case $K = 4$, $A = \{1, 2\}$, $B = \{3, 4\}$ and the utility function $max\{x_1x_3, x_2x_4\}$. (Note that this is the max, not min function)

c) (Take Home) Show that if the consumer follows the above procedure, then it might be that his overall choice cannot be rationalized (For the first stage, you can choose a simple function like $v = min\{w_A, w_B\}$.

Question 3:

An agent makes a binary comparisons of pairs of numbers. His real interest is to maximize the sum $x_1 + x_2$. When he compares (x_1, x_2) and (y_1, y_2) he makes always the right comparison if one of the pairs dominates the other. When this is not the case he might make a mistake. The technology of mistake is characterized by a function $\alpha(G, L)$ with the interpretation that if the gain in one dimension is $G \geq 0$ and the loss in the other dimension is $L \geq 0$, then the probability of mistake is $\alpha(G, L)$.

a) Suggest reasonable and workable assumptions for the function α (such as $\alpha(G, L) \leq 1/2$ for all G and L).

b) Suggest a notion expressing the phrase "agent 1 is more accurate than agent 2".

c) Show that with the notion you defined in b) the probability that three binary comparisons on the triple $(7, 2)$, $(3, 10)$, $(0, 6)$ yields a cycle is smaller for the agent who is more accurate.

d) Show that the probability the binary comparisons will yield cycle on a general triple of pairs is not necessarily smaller for the agent who is more accurate.

e) (Take Home): Show that if agent 1 is characterized by a mistake function $\alpha(G, L)$ and agent 2 is characterized by the mistake function $\lambda\alpha(G, L)$ for $0 < \lambda < 1$, then for a triple of pairs, the probability that the agent 2's answers exhibit a cycle is less than the probability that agent 1's answers exhibit a cycle.