Course: **Microeconomics I, New York University**

Lecturer: **Ariel Rubinstein**

Exam: **Mid-term, October 2012**

Time: **3 hours (no extensions)**

Question 1

A consumer operates in a world with K commodities. He has in mind a list of consumption priorities, a sequence (k_n, q_n) where $k_n \in \{1, \ldots, K\}$ is a commodity and q_n is a quantity. When facing a budget set (p, w) he purchases the goods according to the order of priorities in the list, until his budget is exhausted. (In the case that his money is exhausted during the n'th stage he purchases whatever proportion of the quantity q_n that he can afford).

(i) How does the demand for the k'th commodity responds to the p_k, p_j $(j \neq k)$ and w?

(ii) Suggest an increasing utility function which rationalizes the consumer's behavior.

(iii) Using the utility function you suggested in (ii) prove the Roy equality for this consumer at (p, w) where the consumer exhausts his entire budget while satisfying his n'th goal.

Question 2

Consider a decision maker in the world of lotteries, with $Z = R$ being monetary prizes. The decision maker assigns a number $v(z)$ to each amount of money z. The function v is continuous and increasing. The decision maker evaluates each lottery p according to:

$$U(p) = \alpha[max\{v(z)|z \in supp(p)\}] + (1-\alpha)[\min\{v(z)|z \in supp(p)\}].$$

(a) Characterize the decision makers of this type who are "risk averse".

(b) Show that if two decision makers of this type, with $\alpha = 1/2$, hold the functions v_1 and v_2 and $v_1 \circ v_2^{-1}$ is concave, then decision maker 1 is more risk averse than decision maker 2.

(c) Do at home: Assume that the two decision makers use $\alpha = 1/2$. Is the concavity of $v_1 \circ v_2^{-1}$ a necessary condition for decision maker 1 to be more risk averse than decision maker 2.

Question 3 (Based on Rubinstein (1980))

An individual is comparing pairs of alternatives within a finite set X ($|X| \geq 3$). His comparison yields unambiguous results, such that either x is evaluated to be better than y (denoted $x \to y$) or y is evaluated to be better than x ($y \to x$). A ranking method assigns to each such relation $\to$ (namely, complete, irreflexive and antisymmetric relation) a preference relation $\succsim(\to)$ over X. Consider the following axioms with respect to ranking methods:

Neutrality: "the names of the alternatives are immaterial". (Formally, let σ be a permutation of X and let $\sigma(\to)$ be the relation defined by $\sigma(x)\sigma(\to)\sigma(y)$ iff $x \to y$. Then, $x \succsim (\to)y$ iff $\sigma(x) \succsim (\sigma(\to))\sigma(y)$.

Monotonicity: if $x \succsim (\to)y$, then $x \succsim (\to')y$ where $\to'$, differs from $\to$ only in the existence of one alternative z such that $z \to x$ and $x \to' z$.

Independence: The ranking between any two alternatives depends only on the results of comparisons that involve at least one of the two alternatives.

(i) Define $N_{\to}(x) = |\{z|x \to z\}|$ (the number of alternatives beaten by x). Explain why the scoring method defined by $x \succsim (\to)y$ if $N_{\to}(x) \geq N_{\to}(y)$ satisfies the three axioms.

(ii) For each of the properties, give an example of a ranking method which satsifies the other two properties but not that one.

(iii) Prove that the above scoring methods is the only one that satisfies the three properties. (In the exam you can make do with a proof for a 4-element set X).