Course: **Microeconomics I**, **New York University**

Lecturer: **Ariel Rubinstein**

Exam: **Mid-term**, **October 2011**

Time: **3 hours** (**no extensions**)

Question 1

Let X be a set of alternatives.

Decision maker of type A uses the following choice procedure. He has a subset of "satisfactory alternatives" in mind. Whenever he chooses from a set, then (i) if there are satisfactory elements in the set, he is happy to choose any satisfactory alternative which comes to his mind and (ii) If there are none, he is happy with any of the non-satisfactory alternatives.

A decision maker of type B has in mind a set of strict orderings. Whenever he chooses from a set, he is happy with any alternative that is the maxima of at least one ordering.

(a) Define formally the two types of decision makers as **choice correspondences**.

(b) Show that any decision maker of type A can also be described as a decision maker of type B.

(c) Show that there is a decision maker of type B who cannot be described as a decision maker of type A.

Question 2

A middleman buys and sells K commodities. He is able to transfer goods between markets 1 and 2 where the price vectors are p^1 and p^2 respectively. A transaction is characterized by $(t_1,..,t_K) \in R^K$ where $t_k > 0$ is a transfer of t_k units of commodity k from market 1 to market 2 and $t_k < 0$ is a transfer of $|t_k|$ units of commodity k from market 2 to market 1. Let T be a set of transactions that he can make. Assume that T is compact and convex.

(a) Formulate the middleman problem if he is a maximizer of profits.

(b) Compare the behavior of the middleman for the two pairs of price vectors (p^1,p^2) and (q^1,q^2) which differ only in $p^2_k > q^2_k$ (for one commodity k).

(c) Assume that T is the constraint whereby the total number of all goods that the middleman can transfer cannot exceed 100. What can you say about the solution of the middleman problem? (No formal proof is necessary.)

(d) Assume that 0 is an interior point of T. What is the necessary and sufficient condition for no trade to be optimal? (No formal proof is necessary)

Question 3

A decision maker forms preferences over "cycles". In each period he is in one of the two states in the set $Z = \{A,B\}$. A *cycle* will be an arbitrary finite sequence of elements in Z. The cycle $(z_1,\ldots,z_L)$ is interpreted as an infinite sequence without a beginning and without an end i.e.. $\ldots.(z_1,\ldots,z_L)(z_1,\ldots,z_L)(z_1,\ldots,z_L)\ldots.$.

With this interpretation in mind, we will assume that **preference relations satisfy the following two properties**:

Invariance to Description: $(z_1,\ldots,z_L) \sim (z_2,\ldots,z_L,z_1)$ for all $(z_1,\ldots,z_L)$.

Duplication: $(z_1,\ldots,z_L) \sim (z_1,\ldots,z_L,z_1,\ldots,z_L,\ldots,z_1,\ldots,z_L)$ for all $(z_1,\ldots,z_L)$.

In this question, we will also discuss the following three properties:

Symmetry: $(z_1,\ldots,z_L) \sim (z_{\sigma(1)},\ldots,z_{\sigma(L)})$ for all $(z_1,\ldots,z_L)$ and for any permutation σ.

Strong Monotonicity: $(y_1,\ldots,A,\ldots,y_L) \succ (y_1,\ldots,B,\ldots,y_L)$ for all $(y_1,\ldots.,y_L)$.

Cancellation: $(y_1,\ldots,y_L) \succsim (z_1,\ldots,z_M)$ and $y_L = z_M$ implies that $(y_1,\ldots,y_{L-1}) \succsim (z_1,\ldots,z_{M-1})$

(a) Construct (no proofs are needed) three examples of preferences over cycles, satisfying Invariance to Description and Duplication. One of them should satisfy Symmetry two should **not** satisfy Symmetry.

(b) What do you think about preferences which satisfy Invariance to Description, Duplication, Strong Monotonicity and Cancellation?

(c) State and prove a proposition regarding the set of preferences which satisfy Invariance to Description, Duplication, Symmetry and Strong Monotonicity.