Course: **Microeconomics, New York University**
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Question 1

A producer in a world with two commodities defines a relation D over the set of technologies satisfying the classical assumptions. ZDZ' if for any price vector p, $max_{x \in Z} px \geq max_{x \in Z'} px$.

(a) Explain the logic behind the definition.
(b) Show that ZDZ' iff $Z \supseteq Z'$.
(c) Define another relation E which will be similar to D and will fit to the assumption that the producer maximizes the production of good 1 subject to non-negative profits and determine if the condition in part (b) is satisfied with this relation.

Question 2

A consumer in a two commodity world, operates in the following way:
The consumer holds a preference relation $\succsim_S$ on the space of his consumption bundles. His father holds a preference relation $\succsim_F$ on the space of his son's consumption bundles. Both relations satisfy strong monotonicity, continuity and strict convexity. The father does not allow his son to purchase any bundle which is not as good (from his perspective) as the bundle $(M, 0)$. The son, when he has to choose from a budget set, maximizes his own preferences subject to the constraint imposed by his father. In case he cannot satisfy his father's wish, he feels free to maximize his own preferences.

(a) Show that the behavior of the son is rationalizable.
(b) Show that the preferences which rationalize this kind of behavior are monotonic, but not necessarily continuous or convex (this part you can demonstrate diagrammatically).
(c) Assume that the father's instruction is that given the budget set (p, w) the son will not purchase any bundle which is $\succsim_F$ worse than $(w/p_1, 0)$. The son's behavior is to maximize his preferences subject to satisfying his father's wish. Show that the son's behavior satisfies the Weak Axiom of Revealed Preferences.
(d) **This part is not for the exam, for you to think at home.** Can you write down a preference relation that rationalizes the son's behavior?

Question 3

Let Z be a finite set of prizes and $L(Z)$ be the space of all lotteries with prizes in Z. An individual has an ordering $\succsim$ on the set Z. His mental attitude towards any pair of lotteries p and q is obtained by the following procedure: he samples **once** p and **once** q, and compares between the outcomes he gets. If the realization of p was better than of q he prefers p to q. (He takes a fresh sample of p when he compares p and r, but always looks at the same sample when comparing p and q.) Thus, the binary relation which describes his mental attitude towards the alternatives is summarized by a binary relation $\succsim$ (which obviously extends the ordering on Z).

(a) Discuss the following properties of the relation: completeness, anti-symmetry, transitivity.
(b) Give an example with three lotteries p, q, r such that the probability that there is a violation of transitivity is $1/4$.
(c) Given the ordering on Z, $\succsim$, define a partial relation
pDq if for every z $\sum_{x \succsim z} p(x) \geq \sum_{x \succsim z} q(x)$.
Show that if pDq then the probability that the individual ranks p above q is at least 0.5.