Course: **Microeconomics, New York University**
Lecturer: **Ariel Rubinstein**
Exam: **Mid-term, October 2009**
Time: **3 hours (no extensions)**
Instructions: **Answer the following three questions (each question in a separate exam booklet)**

Question 1

We will say that a choice function C is consistent with the majority vetoes a dictator procedure if there are three preference relations $\succ_1$, $\succ_2$ and $\succ_3$ such that $c(A)$ is the $\succ_1$ maximum unless both $\succ_2$ and $\succ_3$ agree on another alternative being the maximum in A.

(a) Show that such a choice function might not be rationalizable.

(b) Show that such a choice function satisfies the following property:

If $c(A) = a$, $c(A - \{b\}) = c$ for b and c different than a then $c(B) = c$ for all B which contains c and is a subset of $A - \{b\}$.

(c) Show that not all choice functions could be explained by the majority vetoes a dictator procedure.

Question 2:

For any non negative integer n and a number $p \in [0, 1]$ let (n, p) be the lottery which gets the prize $\$n$ with probability p and $\$0$ with probability $1-p$. Let us call those lotteries "simple lotteries". Consider preference relations on the space of simple lotteries.

We say that such a preference relation satisfies Independence if $p \succeq q$ iff $\alpha p \oplus (1-\alpha)r \succeq \alpha q \oplus (1-\alpha)r$ for any $\alpha > 0$, and any simple lotteries p, q, r for which the compound lotteries are also simple lotteries.

Consider a preference relation satisfying the Independence axiom, strictly monotonic in money and continuous in p.

Show that:

(a) (n, p) is monotonic in p for $n > 0$, i.e. for all $p > p'$ $(n, p) \succ (n, p')$

(b) For all n there is a unique $v(n)$ such that $(1, 1) \sim (n, 1/v(n))$

(c) It can be represented with the expected utility formula: that is there is an increasing function v such that $pv(n)$ is a utility function which represents the preference relation.

Question 3:

An economic agent is both a producer and a consumer. He has a_0 units of good 1. He can use some of a_0 to produce commodity 2. His production function f satisfies monotonicity, continuity, strict concavity. His preferences satisfy monotonicity, continuity and convexity. Given he uses a units of commodity 1 in production he is able to consume the bundle $(a_0 - a, f(a))$ for $a \leq a_0$. The agent has in his "mind" three "centers":

The pricing center declares a price vector (p_1, p_2).

The production center takes the price vector as given and he operates according to one of the following two rules

Rule 1: he maximizes profits: $p_2 f(a) - p_1 a$.

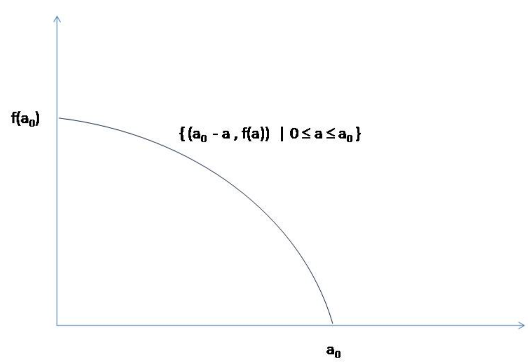
Rule 2: he maximizes production subject to the constraint that he does not make any losses, i.e. $p_2 f(a) - p_1 a \geq 0$.

The output of the production center is a consumption bundle.

The consumption center takes $(a_0 - a, f(a))$ as endowment, and finds the optimal consumption allocation that he can afford according to the prices declared by the pricing center.

The prices declared by the pricing center are chosen to create harmony between the other two centers in the sense that the consumption center finds the outcome of the production center's activity, $(a_0 - a, f(a))$, optimal given the announced prices.

Hint: The set of all possible consumption bundles is bounded by $\{(a_0 - a, f(a)) | 0 \leq a \leq a_0\}$



(a) Show that under Rule 1, the economic agent consumes the bundle $(a_0 - a*, f(a*))$ which maximizes his preferences.

(b) What is the economic agent's consumption with Rule 2?

(c) State and prove a general conclusion about the comparison between the behavior of two individuals, one whose production center operates with Rule 1 and one whose production center activates Rule 2.