**Course:** **Microeconomics, New York University**
**Lecturer:** **Ariel Rubinstein**
**Exam Solutions:** **Mid-term, October 2009**

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**Question 1**

*We will say that a choice function $C$ is consistent with the majority vetoes a dictator procedure if there are three preference relations $\succ_1$, $\succ_2$ and $\succ_3$ such that $c(A)$ is the $\succ_1$ maximum unless both $\succ_2$ and $\succ_3$ agree on another alternative being the maximum in $A$.*

*(a) Show that such a choice function might not be rationalizable.*

We will show that the choice function violates property $\alpha$. Consider the following preferences on $a, b, c, d$:

$$a \succ_1 b \succ_1 c$$
$$b \succ_2 a \succ_2 c$$
$$c \succ_3 b \succ_3 a$$

According to these preferences

$$C(\{a,b,c\}) = a$$
$$C(\{a,b\}) = b$$

*(b) Show that such a choice function satisfies the following property:*
*If $c(A) = a$, $c(A - \{b\}) = c$ for $b$ and $c$ different than $a$ then $c(B) = c$ for all $B$ which contains $c$ and is a subset of $A - \{b\}$.*

Claim: $a$ is $\succ_1$ maximal in $A$. Assume not for contradiction. Then, $a$ must be $\succ_2$ and $\succ_3$ maximal in $A$. But since $A - \{b\}$ is a subset of $A$, $a$ must be $\succ_2$ and $\succ_3$ maximal there, too. But according to the majority veto dictator rule it must also be chosen in $A - \{b\}$ which contradicts $c(A - \{b\}) = c$.

By claim, and $A - \{b\} \subset A$, we know that $a$ is $\succ_1$ maximal in $A - \{b\}$. Since $c(A - \{b\}) = c$, we know that $c$ must be $\succ_2$ and $\succ_3$ maximal in $A - \{b\}$. (Otherwise $a$ would be chosen in $A - \{b\}$.) But $c$ must be maximal $\succ_2$ and $\succ_3$ in any $B$ which contains $c$ and is a subset of $A - \{b\}$ . Then $c$ must be chosen in any of these subsets.

*(c) Show that not all choice functions could be explained by the majority vetoes a dictator procedure.*

Any choice function satisfying $C(\{a,b,c,d\}) = a$, $C(\{a,c,d\}) = c$, and $C(\{a,c\}) = a$ violates the property in part (2) and thus cannot be explained by the majority vetoes a dictator procedure.

**Question 2:**

*For any non negative integer $n$ and a number $p \in [0,1]$ let $(n,p)$ be the lottery which gets the prize \$$n$ with probability $p$ and \$0 with probability $1-p$. Let us call those lotteries "simple lotteries". Consider preference relations on the space of simple lotteries.*

*We say that such a preference relation satisfies Independence if $p \succeq q$ iff $\alpha p \oplus (1-\alpha)r \succeq \alpha q \oplus (1-\alpha)r$ for any $\alpha > 0$, and any simple lotteries $p, q, r$ for which the compound lotteries are also simple lotteries.*

*Consider a preference relation satisfying the Independence axiom, strictly monotonic in money and continuous in $p$.*

*Show that:*

*(a) $(n,p)$ is monotonic in $p$ for $n > 0$, i.e. for all $p > p'$ $(n,p) \succ (n,p')$*

Observation 1: By monotonicity, $(n,1) \succ (m,1)$ for all $m < n$.
Observation 2: For all $n$, $(n,0) \sim (0,1)$ since both lotteries give 0 w.p. 1.

Proof of Claim:
By observation 1 and 2, $(n,1) \succ (n,0)$.
By independence $p'/p(n,1) \oplus (1-p'/p)(n,1) \succ p'/p(n,0) \oplus (1-p'/p)(n,1)$.
$\Rightarrow (n,1) \succ (n,p'/p)$
Using independence again, $p(n,1) \oplus (1-p)(n,0) \succ p(n,p'/p) \oplus (1-p)(n,0)$
$\Rightarrow (n,p) \succ (n,p')$

*(b) For all $n$ there is a unique $v(n)$ such that $(1,1) \sim (n, 1/v(n))$*

By observations above for $n > 1$, $(n,1) \succ (1,1) \succ (n,0)$

Since $(n,p)$ is continuous, and monotonic in $p$, there exists a unique $p_n$ such that $(1,1) \sim (n,p_n)$.

Denote $v(n)$ such that $v(n) = 1/p_n$, and $v(0) = 0$, and naturally $v(1) = 1$

*(c) It can be represented with the expected utility formula: that is there is an increasing function $v$ such that $pv(n)$ is a utility function which represents the preference relation.*

Claim : For $n > m$, $v(n) > v(m)$
By monotonicity in money, $(n,1) \succ (m,1)$.
By independence $(n,1/v(n)) \sim (1,1) \succ (m,1/v(n))$
$\Rightarrow (m,1/v(m)) \succ (m,1/v(n))$.
By monotonicity in $p$, $1/v(m) > 1/v(n)$

Now lets check that $u(n,p) = v(n)p$ represents preferences over these lotteries.
Note that, $(n,1/v(n)) \sim (m,1/v(m))$.
By independence $(n, v(m)/v(n)q) \sim (m,q)$
Then $(n,p)$ relates to $(m,q)$ like $p$ relates to $v(m)/v(n)q$.
Thus $(n,p) \succ (m,q)$ iff $v(n)p > v(m)q$

**Question 3:**

*An economic agent is both a producer and a consumer. He has $a_0$ units of good 1. He can use some of $a_0$ to produce commodity 2. His production function $f$ satisfies monotonicity, continuity, strict concavity. His preferences satisfy monotonicity, continuity and convexity. Given he uses a units of commodity 1 in production he is able to consume the bundle $(a_0 - a, f(a))$ for $a \leq a_0$. The agent has in his "mind" three "centers":*

*The pricing center declares a price vector $(p_1, p_2)$.*

*The production center takes the price vector as given and he operates according to one of the following two rules*

*Rule 1: he maximizes profits: $p_2 f(a) - p_1 a$.*

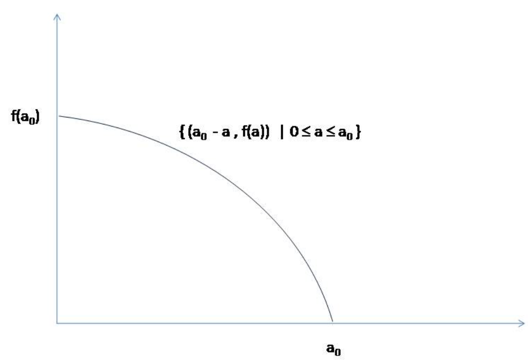
*Rule 2: he maximizes production subject to the constraint that he does not make any losses,, i.e. $p_2 f(a) - p_1 a \geq 0$.*

*The output of the production center is a consumption bundle.*

*The consumption center takes $(a_0 - a, f(a))$ as endowment, and finds the optimal consumption allocation that he can afford according to the prices declared by the pricing center.*

*The prices declared by the pricing center are chosen to create harmony between the other two centers in the sense that the consumption center finds the outcome of the production center's activity, $(a_0 - a, f(a))$, optimal given the announced prices.*

*Hint: The set of all possible consumption bundles is bounded by $\{(a_0 - a, f(a)) | 0 \leq a \leq a_0\}$*



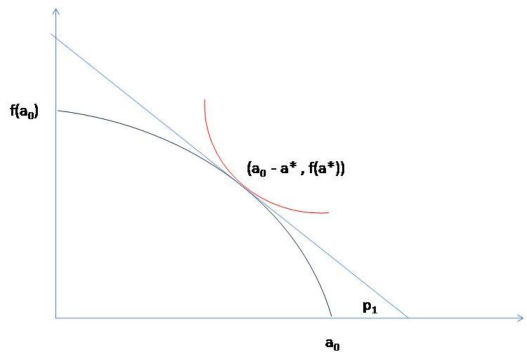

*(a) Show that under Rule 1, the economic agent consumes the bundle $(a_0 - a*, f(a*))$ which maximizes his preferences.*

The solution corresponds to the point on the production possibility set where preferences are maximized.

Since the production possibility set is strictly convex, and preferences are convex we know that there is a unique maximum.

Now choose a price vector such that the price line is tangent to this set and the indifference line exactly at the maximum.

Figure 1: Solution with Rule 1

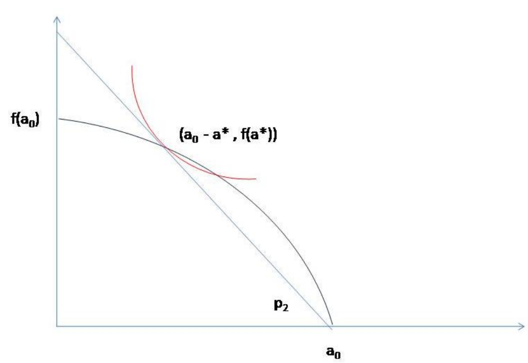



By construction, profit is maximized given prices, and preferences are maximized at the intersection point for given prices and endowment point $(a_0 - a*, f(a*))$.

*(b) What is the economic agent's consumption with Rule 2?*

The economic agent chooses $(a_0 - a*, f(a*))$ with maximal $a*$ subject to the constraint that preferences are maximized at this point when we take the line connecting this point to $(a_0, 0)$ as the price line. By construction, production is maximized here

Figure 2: Solution with Rule 2



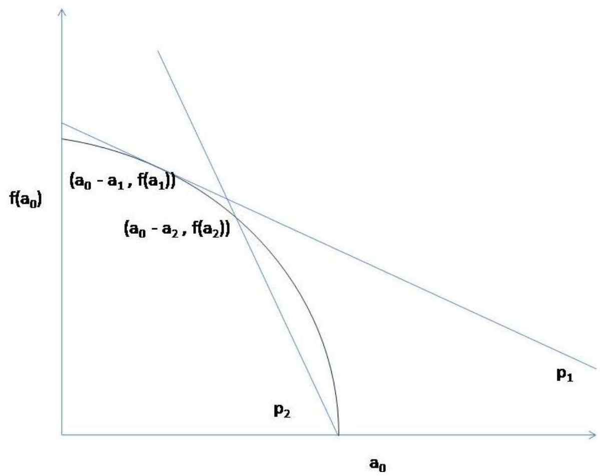

subject to the constraint that there are no losses with the given prices. Also the point is chosen to guarantee that the consumer preferences are maximized at the budget set with the same prices.

*(c) State and prove a general conclusion about the comparison between the behavior of two individuals, one whose production center operates with Rule 1 and one whose production center activates Rule 2.*

Claim: Individual using Rule 2 will always produce more, i.e. for $a_1, p^1$ and $a_2, p^2$ denoting the solutions under Rule 1 and Rule 2, $f(a_1) \leq f(a_2)$.

Assume for contradiction that $a_1 > a_2$. This means that the solution with Rule 2 is strictly to the right of the solution with Rule 1. Since $f$ is strictly concave and

Figure 3: Graph showing why $a_1$ can't be greater than $a_2$




monotonic, if solution with Rule 2 is to the right of solution with Rule 1, we must have $\frac{p_1^1}{p_2^1} < \frac{p_1^2}{p_2^2}$.

Note that $(a_0 - a_1, f(a_1))$ affordable (and strictly interior) in the budget set defined by $p^2$. Also $(a_0 - a_2, f(a_2))$ is affordable (and strictly interior) in the budget set defined by $p^1$.

$\Rightarrow (a_0 - a_2, f(a_2)) \succ (a_0 - a_1, f(a_1))$

$\Rightarrow (a_0 - a_1, f(a_1)) \succ (a_0 - a_2, f(a_2))$

which is a contradiction.