**Course:** **Microeconomics, New York University**
**Lecturer:** **Ariel Rubinstein**
**Exam:** **Mid-term, October 2008**
**Time:** **3 hours (no extensions)**
**Instructions:** **Answer the following three questions, each question in a seperate exam book.**

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**Question 1**

A decision maker has a preference relation over the pairs $(x_{me}, x_{him})$ with the interpretation that $x_{me}$ is an amount of money he will get and $x_{him}$ is the amount of money another person will get. Assume that

(i) for all $(a, b)$ such that $a > b$ the decision maker strictly prefers $(a, b)$ over $(b, a)$.

(ii) if $a' > a$ then $(a', b) \succ (a, b)$.

The decision maker has to allocate $M$ between him and another person.

(a) Show that these assumptions guarantee that he will never allocate to the other person more than he gives to himself

(b) Assume (i), (ii) and

(iii) The decision maker is indifferent between $(a, a)$ and $(a-\varepsilon, a+4\varepsilon)$ for all $a$ and $\varepsilon > 0$.
Show that nevertheless he might allocate the money equally.

(c) Assume (i), (ii), (iii) and

(iv) The decision maker's preferences are also differentiable.

Show that in this case, he will allocate to himself (strictly) more than to the other.

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**Question 2** (based on work of Kfir Eliaz and Ariel Rubinstein)

Let $X$ be a (finite) set of alternatives. Given any choice problem $A$ (where $|A| \geq 2$), the decision maker chooses a set $D(A) \subseteq A$ of **two** alternatives which he wants to examine more carefully before making the final decision.

The following are two properties of $D$:

**A1:** If $a \in D(A)$ and $a \in B \subset A$ then $a \in D(B)$.

**A2:** If $D(A) = \{x, y\}$ and $a \in D(A - \{x\})$ for some $a$ different than $x$ and $y$, then $a \in D(A - \{y\})$.

Answer the following four questions. A full proof is required only for the last question:

(a) Find an example of a $D$ function which satisfies both A1 and A2.

(b) Find a function $D$ which satisfies A1 and not A2 .

(c) Find a function $D$ which satisfies A2 and not A1.

(d) (*) Characterize the set of $D$ functions which satisfy both axioms .

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**Question 3**

An economic agent has to choose between projects. The outcome of each project is uncertain. It might yield a failure or one of $K$ "types of success". Thus, each project $z$ can be described by a vector of $K$ non-negative numbers, $(z_1, ..., z_K)$ where $z_k$ stands for the probability that the project success will be of type $k$.

Let $Z \subset \Re^K_+$ be the set of feasible projects. Assume $Z$ is compact, convex and satsifies "free disposal".

The decision maker is an Expected Utility maximizer.

Denote by $u_k$ the vNM utility from the $k$–th type of success, and attach 0 to failure. Thus the decision maker chooses a project (vector) $z \in Z$ in order to maximize $\sum z_k u_k$.

(a) First, formalize the decision maker's problem. Then, formalize (and prove) the claim: If the decision maker suddenly values type $k$ success higher than before, he would choose a project assigning a higher probability to $k$.

(b) Apparently, the decision maker realizes that there is an additional uncertainty. The world may go "one way or another". With probability $\alpha$ the vNM utility of the $k$'th type of success will be $u_k$ and with probability $1 - \alpha$ it will $v_k$. Failure remains 0 in both contingencies.

First, formalize the decision maker's new problem. Then, formalize (and prove) the claim: Even if the decision maker would obtain the same expected utility, would he have known in advance the direction of the world, the existence of uncertainty makes him (at least weakly) less happy.