Course: Microeconomics, New York University
Lecturer: Ariel Rubinstein
Exam: Mid-term, October 2007
Time: 3 hours (no extensions)
Instructions: Answer the following three questions. The two "starred" parts are more difficult and it is recommended you attempt them only after you have completed the other parts.

Question 1

A consumer in a world of K commodities maximizes the utility function $u(x) = \sum_k x_k^2$.

(a) Calculate the consumer's demand function (whenever it is uniquely defined).

(b) Give another preference relation (not just a monotonic transformation of u) which induces the same demand function.

(c) For the original utility function u, calculate the indirect preferences for $K = 2$. What is the relationship between the indirect preferences and the demand function? (It is sufficient to answer for the domain where $p_1 < p_2$.)

(d) Are the preferences in (a) differentiable (according to the definition given in class)?

Question 2

Let X be a set and C be a choice correspondence defined on all non-empty subsets of X. We say that C satisfies Path independence (PI) if for every two disjoint sets A and B, we have $C(A \cup B) = C(C(A) \cup C(B))$. We say that C satisfies Extension (E) if $x \in A$ and $x \in C(\{x,y\})$ for every $y \in A$ implies that $x \in C(A)$ for all sets A.

(a) Interpret PI and E.

(b) Show that if C satisfies both PI and E, then there exists a binary relation $\succsim$ that is complete, reflexive and satisfies $x \succ y$ and $y \succ z$ implies $x \succ z$, such that $C(A) = \{x \in A \mid \text{for no } y \in A \text{ is } y \succ x\}$.

(c*) Give one example of a choice correspondence: either one satisfying PI but not E, or one satisfying E but not PI.

Question 3

Identify a professor's lifetime with the interval $[0,1]$. There are $K+1$ academic ranks, $0,..,K$. All professors start at rank 0 and eventually reach rank K sooner or later. Define a career as a sequence $t = (t_1,\ldots,t_K)$ where $t_0 = 0 \leq t_1 \leq t_2 \leq\ldots\leq t_K \leq 1$ with the interpretation that t_k is the time of the k'th promotion. (Note that a professor can receive multiple promotions at the same time.) Denote by $\succsim$ the professor's preferences on the set of all possible careers.

For any $\epsilon > 0$ and for any career t such that $t_K \leq 1-\epsilon$ define $t+\epsilon$ to be the career $(t+\epsilon)_k = t_k + \epsilon$ (all promotions are delayed by ϵ).

Here are two properties of the professor's preferences:

Monotonicity: For any two careers t and s, if $t_k \leq s_k$ for all k then $t \succsim s$ and if $t_k < s_k$ for all k then $t \succ s$.

Invariance: For every $\epsilon > 0$ and every two careers t and s for which $t+\epsilon$ and $s+\epsilon$ are well defined, $t \succsim s$ iff $t+\epsilon \succsim s+\epsilon$.

(a) Formulate the set L of careers in which a professor receives all K promotions at the same time. Show that if $\succsim$ satisfy continuity and monotonicity, then for every career t there is a career $s \in L$ such that $s \sim t$.

(b) Show that any preference which is represented by the function $U(t) = -\sum \Delta_k t_k$ (for some $\Delta_k > 0$) satisfies Monotonicity, Invariance and Continuity.

(c*) One professor evaluates a career by the maximum length of time he had to wait for a promotion, and the smaller this number the better. Show that these preferences cannot be represented by the utility function described in (b).