Course: **Microeconomics, New York University**
Lecturer: **Ariel Rubinstein**
Exam: **Mid-term, October 2006**
Time: **3.5 hours (no extensions)**
Instructions: Answer the following three questions in three seperate exam-books.

Problem 1. Consider a consumer in a world of 2 commodities who has to make choices from budget sets parameterized by (p,w,c) where p is a vector of prices, w is a wealth level and c is a limit on consumption of good 1. That is, in his world, a choice problem is a set of the form $B(p,w,c) = \{x \mid px \leq w$ and $x_1 \leq c\}$. Denote by $x(p,w,c)$ the choice of the consumer from $B(p,w,c)$.

(a) Assume $px(p,w,c) = w$ and that $x_1 = \min\{0.5w/p_1, c\}$. Show that this behavior is consistent with the assumption that demand is derived from a maximization of some preference relation.

(b) Assume that $px(p,w,c) = w$ and that $x_1(p,w,c) = \min\{0.5c, w/p_1\}$. Show that this consumer's behavior is **inconsistent** with preference maximization.

(c) Assume that the consumer makes his choice by maximizing the utility function $u(x)$. Denote the indirect utility by $V(p,w,c) = u(x(p,w,c))$. Assume that V is "well-behaved". Show how one could derive the demand function from the function V in the range where $\partial V/\partial c(p,w,c) > 0$.

Problem 2. (based on Rubinstein and Salant (2006)). Let X be a grand finite set. Consider a model where a choice problem is a pair (A,a) where A is a subset of X and $a \in A$ is interpreted as a default.

A decision maker's behavior can depend on the default point as well and thus is described by a function $c^*(A,a)$ which assigns an element in A to each choice problem (A,a).

Assume that c^* satisfies the following two properties:

Default bias: If $c^*(A,a) = x$, then $c^*(A,x) = x$.

Extended IIA: If $c^*(A,a) = x$ and $x \in B \subseteq A$, then $c^*(B,a) = x$.

(a) Give two examples of a function c^* which satisfy the above two properties.

(b) Define a relation $x \succ y$ if $c^*(\{x,y\},y) = x$. Show that the relation is asymmetric and transitive.

(c) Explain why the relation $\succ$ may be incomplete.

(d-bonus) Define a choice correspondence $C(A) = \{a|$ there exists $x \in A$ such that $c^*(A,x) = a\}$ that is, $C(A)$ is the set of all elements in A which are chosen given some default alternative. Show that $C(A)$ is the set of all $\succ$ maximal elements and interpret this result.

Problem 3. Consider a world with balls of K different colors. Define a *bag* to be a vector $x = (x_1, .., x_K)$, where x_k is a non-negative integer indicating the number of balls of color k in the bag. Define $n(x) = \sum x_k$ (the number of balls in the bag x). Let X be the set of all bags.

(a) Show that any preference relation over X which is represented by $U(x) = \sum_k x_k v_k / n(x)$ (for some vector of numbers (v_k)) satisfies the following two axioms:

(A1) For any $x \in X$ and for any natural number λ, $x \sim \lambda x$.

(A2) For any $x, y \in X$ such that $n(x) = n(y)$ and for any $z \in X$,
$x \succsim y$ iff $x + z \succsim y + z$.

(b) Suggest a context in which it makes sense to assume those two axioms.

(c) Find a preference relation that satisfies the two axioms and which cannot be represented in the form suggested in (a) (prove it).
