Course: **Microeconomics, New York University**
Lecturer: **Ariel Rubinstein**
Exam: **Mid-term, October 2004**
Time: **3 hours (no extensions)**
Instructions: **Answer the following three questions. Be concise and accurate as possible.**

Problem 1

A decision maker has in mind a binary relation $\succ$ on the set X with the interpretation that $a \succ b$ means that when he compares a with b he chooses a. Assume that $\succ$ is complete and asymmetric but not necessarily transitive.

Each of the alteratives in X has a distinct ID number.

The following is a "forward" procedure to choose from any finite choice set A: Use $\succ$ to compare the two alternatives with the **lowest** IDs. The "loser" leaves the set and continue with the procedure continues with the "winner" and the rest of the set and so on until only one element remains. This element is chosen.

The following is a "backward" procedure to choose from any finite choice set A: Use $\succ$ to compare the two alternatives with the **highest** IDs. The "loser" leaves the set and the procedure continues with the "winner" and the rest of the set and so on until only one element remains. This element is chosen.

a) Do the two choice functions which were constructed by these procedures satisfy condition $*$?

b) Give an example where the two procedures yield different outcomes?

c) Define a condition under which the two procedures will always yield the same outcome.

Problem 2

Consider a classical consumer in the world of two goods Prices are non-linear. Let $p_2 = 1$ and $p(q)$ be the price of one unit of commodity 1 if he purchases q units of that commodity. Assume that the function $p(q)q$ is a strictly increasing continuous concave function. The consumer's wealth is w.

a) Present the consumer's problem.

b) Suppose that for a consumer who maximizes a utility function u there is a unique optimal choice of the first commodity, $x_1^* > 0$. Show that if instead of the price schedule $p(q)$ he faces the linear price schedule $p'(q) \equiv p(x_1^*)$, he would not purchase more than x_1^* of commodity 1.

c) Define $e(p,u)$ (the expenditure function) to be the minimal w needed by the consumer, when facing the price schedule p, to attain the utility level u. Show that:

i) if $p'(q) \geq p(q)$ for all q then $e(p',u) \geq e(p,u)$

ii) $e(\lambda p + (1-\lambda)p', u) \geq \lambda e(p,u) + (1-\lambda)e(p',u)$

($\lambda p + (1-\lambda)p'$ is the price schedule $(\lambda p + (1-\lambda)p')(q) = \lambda p(q) + (1-\lambda)p'(q)$)

Problem 3

Let X be the space of finite sequences of non-negative numbers. A sequence $(x_1,..,x_K) \in X$ is interpreted as a stream of sums of money which a shopkeeper earns. The following are several properties of his preferences on X:

A1: The shopkeeper has difficulty in dealing with a large number of transactions. He prefers to get the money in one lump sum, i.e., the single number sequence $(\sum x_k)$ on the sequence of $(x_1,..,x_K)$.

A2: He is indifferent between $(x_1,..,x_K)$ and any permutation of the sequence.

A3: More is better: He strictly prefers the sequence $(x_1,..,x_{K+1})$ to the sequence $(x_1,..,x_K)$ for any $x_{K+1} > 0$.

A4: If he prefers the sequence x to the sequence y then he prefers the sequence which combines x and z over the sequence which combines y and z.

A5: A receipt of 0 does not hurt. He is always indifferent between $(x_1,..,x_K,0)$ and $(x_1,..,x_K)$.

a) Show that for any convex increasing function v with $v(0) = 0$ the utility function $u(x_1,\ldots,x_K) = \sum_{k=1,..,K} v(x_k)$ represents preferences satisfying A1-2-3-4-5.

b) For any of the five properties, suggest a preference relation which does not satisfy that property but does satisfy the other four. (I know it will take you too much time to find five examples, so don't worry, find as many examples as you can and don't provide proofs)).