Course: **Microeconomics, New York University**

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Time: **3 hours (no extensions)**

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Problem 1

In a world with K commodities, consider a consumer's preferences which are represented by the utility function $u(x_1, x_2, .., x_K) = \sum_{k=1,..,K} x_k$.

(a) Calculate the consumer's demand function (whenever it is well defined).

The consumers problem is:

choose $x = (x_1, x_2, \ldots x_k)$

to maximize $u(x) = \sum_{k=1,..,K} x_k$

subject to $px \leq w$

we have a welldefined demand function when $p_i = \min p_k$ for a unique i.

$x_k(p,w) = \frac{w}{p_i}$ for $i = k$ and $x_k(p,w) = 0$ otherwise.

(b) Calculate the indirect utility function, $v(p,w)$.

$v(p,w) = u(x(p,w)) = \sum_{k=1,..,K} x_k(p,w) = x_i(p,w) = \frac{w}{p_i}$

(c) Verify Roy's Equality.

for commodity i we have: $-\frac{\frac{\delta v(p,w)}{\delta p_i}}{\frac{\delta v(p,w)}{\delta w}} = -\frac{-\frac{w}{p_i^2}}{\frac{1}{p_i}} = \frac{w}{p_i} = x_i(p,w)$

for all other commodities k we have: $-\frac{\frac{\delta v(p,w)}{\delta p_k}}{\frac{\delta v(p,w)}{\delta w}} = -\frac{0}{\frac{1}{p_i}} = 0 = x_k(p,w)$.

(d) Calculate the expenditure function $e(p,u)$.

minimize w

subject to $v(p,w) \geq u$

that is

minimize w subject to to $\frac{w}{p_i} = u$

so we have $e(p,u) = p_i u$.

Problem 2

In this question, we will consider a consumer which behaves differently than the classical consumer we talked about in class. Once again we consider a world with K commodities. The consumer's choice will be from budget sets. The consumer has in mind a preference relation which satisfies continuity, monotonicity and strict convexity and for simplicity assume it is represented by a utility function u.

The consumer aims to maximize utility as long as he does not obtain the utility level u^0. If the budget set allows him to obtain this level of utility he chooses the bundle in the budget set with the highest quantity of commodity 1 subject to the constrain that his utility is at least u_0.

a) Formulate the consumer's problem.

if there does not exist any x such that $px \leq w$ and $u(x) \geq u^0$

choose $x = (x_1, x_2, \ldots x_k)$

to maximize $u(x)$

subject to $px \leq w$

otherwise

choose $x = (x_1, x_2, \ldots x_k)$

to maximize x_1

subject to $px \leq w$ and $u(x) \geq u^0$.

b) Show that the consumer's procedure yields a unique bundle.

Suppose the consumer's procedure would yield two bundles $x^{\#}$ and x^*.

First assume that there does not exist any x such that $px \leq w$ and $u(x) \geq u^0$. So we have $px^{\#} \leq w$ and $px^* \leq w$ and consequently also $p(\frac{1}{2}x^{\#} + \frac{1}{2}x^*) \leq w$. However, by the strict convexity of preferences we have that $u(\frac{1}{2}x^{\#} + \frac{1}{2}x^*) > u(x^{\#})$ a contradiction.

So we must have that there exists an x such that $px \leq w$ and $u(x) \geq u^0$. We cannot have $x_1^{\#} \neq x_1^*$ as in this case only one of them would be chosen. So we must have that $x_1^{\#} = x_1^*$. By strict convexity we have $u(\frac{1}{2}x^{\#} + \frac{1}{2}x^*) > \min(u(x^{\#}), u(x^*)) \geq u^0$. By continuity there exists an ε Ball around $\frac{1}{2}x^{\#} + \frac{1}{2}x^*$ such that $u(x') > u^0$ for all x' in that Ball. In particular there exists a x'' in that Ball with $x_1'' > x_1^{\#}$ and $x_k'' \leq x_k^{\#}$ such that $px'' \leq w$. So the consumer should not have chosen $x^{\#}$.

c) Is this demand procedure rationalizable?

Yes, the preferences represented by the function u' rationalize the demand of the consumer:

$u'(x) = u(x)$ if $u(x) \leq u^0$ and $u'(x) = u^0 + x_1$ otherwise.

d) Does the demand function satisfy the Walras Law?

Yes, suppose the consumer would in optimum only spend $px(p,w) := w' < w$. As $u'(x)$ is strictly increasing in x_1 we have that $u'(x) < u'(x_1 + \frac{w-w'}{p_1}, x_2, \ldots, x_K)$ for all x. So, to maximize his utility, the consumer has to spend his entire income.

e) Show that in the range of (p,w) for which there a feasible bundle yielding utility of at least u^0 the consumer's demand function for commodity 1 is decreasing in p_1 and increasing in w.

Suppose $p_1 \geq p_1'$ and $p_k' = p_k$ for $1 \neq k$ and $w \leq w'$. Observe that as good one gets cheaper and/or income larger the consumer can afford to buy more of x_1. And consuming more of x_1 increases the consumer's utility u' :

$u'(x(p',w')) \geq u^0 + x_1(p,w) + \varepsilon = u'(x(p,w)) + \varepsilon$. Where $\varepsilon := \frac{w'-p'x}{p_1'}$. If we have $p_1 > p_1'$ or $w < w'$ we have $\varepsilon > 0$ or equivalently $u'(x(p',w')) > u'(x(p,w))$. Now observe that for $u(x) \geq u^0$ the utility $u'(x)$ only increases when x_1 increases. So for $u'(x(p',w')) > u'(x(p,w))$ to hold we have to have $x_1(p',w')) > x_1(p,w)$ and consequently $x_1(p,w)$ is decreasing in p_1 and increasing in w.

f) Is the demand function continuous?

Yes. The utility function u' is only discontinuous at bundles x for which $u(x) = u^0$. Therefore we have that at any (p,w) for which there is no x in the budget set such that $u(x) \geq u^0$ the demand for x_1 is continuous. On the other hand for any (p,w) such that there exists an x_1 such that $p_1 x_1 \leq w$ and $u(x_1, 0, \ldots, 0) > u^0$ we have that $x_1(p,w) = \frac{w}{p_1}$ a continuous function.

So we only need to consider the case that the consumer chooses a bundle x for which

$u(x) = u^0$. So take some sequence $(p^n, w^n) \to (p, w)$ with $x(p^n, w^n) \to x' \neq x(p, w)$ and $u(x(p, w)) = u^0$. As $p^n x(p^n, w^n) \leq w^n$ for all n we have that $px' \leq w$ so x' is affordable at (p, w) and consequently we must have $u'(x') < u'(x(p, w))$

Case 1: $u'(x') < u^0$. This implies $u'(x') = u(x)$. As u is continuous we have that there exist ε Balls around x' and $x(p, w)$ such that for all x'' in the first Ball and all x^* in the second Ball we have $u(x'') < u(x^*)$ In particular for n large enough we have that $x(p^n, w^n)$ is in the Ball around x'' but as $(p^n, w^n) \to (p, w)$ we can find $x^{**} \ll x^*$ in the second Ball such that $p^n x^{**} \leq w^n$. A contradiction as x^{**} affordable at (p^n, w^n) but $u(x^{**}) > u(x(p^n, w^n))$.

Case 2: $u^0 \leq u(x') < u(x(p, w)) = u^0 + x_1(p, w)$. There exists a bundle x'' such that $px'' < w$, $u(x'') \geq u^0$ and $x_1'' > x_1'$. Now let $\varepsilon := \frac{x_1'' - x_1'}{3}$. For large enough n we have that $p^n x'' \leq w^n$. And as $x(p^n, w^n) \to x'$ implies $x_1(p^n, w^n) \to x_1'$ we have for large n that $u(x(p^n, w^n)) < u(x') + \varepsilon$. So for these large enough n there is bundle that is affordable at p^n, w^n namely the bundle x'' that has $u'(x'') > u(x') + \varepsilon > u(x(p^n, w^n))$.

Problem 3

A decision maker has a preference relation $\succsim$ over the space of lotteries $L(Z)$ with a set of prizes Z.

He knows at Sunday 1 that on Monday it will be revealed whether he has to choose between L_1 and L_2 (probability $1 > \alpha > 0$) or between L_3 and L_4 (probability $1 - \alpha$). Then he will make the choice.

Let us compare between two possible approaches the decision maker may take:

Approach 1: He delays his decision to Monday ("why to bother with the decision now while I can make mind tomorrow"..:)).

Approach 2: He makes a contingent decision on Sunday regarding what he will do on Monday, that is he instructs his machine/agent/himself what to do if he faces the choice between L_1 and L_2 and what to do if he faces the choice between L_3 and L_4 ("On Monday morning I will be so busy"...:)).

a) Formulate approach 2 as a choice between lotteries.

$L_{1,3}$ with probability α get lottery L_1 with probability $1 - \alpha$ get L_3

$L_{1,4}$ with probability α get lottery L_1 with probability $1-\alpha$ get L_4

$L_{2,3}$ with probability α get lottery L_2 with probability $1-\alpha$ get L_3

$L_{2,4}$ with probability α get lottery L_2 with probability $1-\alpha$ get L_4.

b) Show that if the preferences of the decision maker satisfy the independence axiom his choice in approach 2 will be always the same as under approach 1.

Let $L_{i,k}$ be the lottery the decision maker chooses under approach two.

that is

$L_{i,k} \succsim L_{l,m}$ for all l, m

in particular

a) $L_{i,k} \succsim L_{i,m}$ for $m \neq k$

and

b) $L_{i,k} \succsim L_{l,k}$ for $l \neq i$

using the independence axiom this holds true if and only if

a') $L_k \succsim L_m$

and

b') $L_i \succsim L_l$

So using approach 1 the decision maker will chose L_k when faced with the choice from amongst L_k and L_m, he will chose L_i when he is faced with the choice from L_i and L_l.

To see that choosing L_k when faced with the choice from amongst L_k and L_m, choosing L_i when he is faced with the choice from L_i and L_l implies choosing $L_{i,k}$ under approach one, observe in addition to the arguments above that form a) $L_{i,k} \succsim L_{i,m}$ for $m \neq k$ b) $L_{i,k} \succsim L_{l,k}$ for $l \neq i$ we can imply by the independence axiom that $L_{i,m} \succsim L_{l,m}$ and by transitivity: $L_{i,k} \succsim L_{l,m}$.

c) Give an example for a preference relation for which the two approaches yield different outcomes!

We have 8 lotteries to consider:

$L_{1,3}$, $L_{1,4}$,$L_{2,3}$, $L_{2,4}$, L_1, L_2 L_3, L_4.

Define the (complete and transitive) preferences on $L(Z)$ by:

$L_1 \succ L_2 \succ L_4 \succ L_{1,3} \succ L_3 \succ L_{1,4} \succ L_{2,3} \succ L_{2,4}$.

These preferences do not satisfy the independence axiom.

According to approach 1 the decision maker will chose L_1 when facing the choice between L_1 and L_2, he will choose L_4 when facing the choice between L_3 and L_4. But if he where to instruct a machine to choose for him he would instruct this machine to pick L_1 and L_3 for him.