Course: Econ 501, Princeton University
Lecturer: Ariel Rubinstein
Exam: Mid-term, October 2002
Time: 3 hours (no extensions)
Instructions: Answer the following two questions. Be concise and accurate.

Problem 1

Consider a consumer with a preference relation in a world of two goods: *X* (an aggregated consumption good) and *M* ("membership in a club", for example), which can be consumed or not. In other words, the consumption of *X* can be any non-negative real number while the consumption of *M* must be either 0 or 1.

Assume that consumer preferences are strictly monotonic, continuous and satisfy property E: For every x there is y such that $(y,0) \succ (x,1)$ (that is, there is always some amount of money which can compensate for the loss of membership).

■A) Show that any consumer's preference relation can be represented by a utility function of the type

$$u(x,m) = \begin{cases} x & if \;\; m = 0 \\ x + g(x) & if \;\; m = 1 \end{cases}$$

■B) (Less easy) Show that the consumer's preference relation can also be represented by a utility function of the type $u(x,m) = \begin{cases} f(x) & if \;\; m = 0 \\ f(x) + v & if \;\; m = 1 \end{cases}$

■C) Explain why continuity and strong monotonicity (without E) are not sufficient for A.

■D) Characterize the consumer's demand function.

■E) Characterize the consumer's indirect utility function. Show that one can derive the demand for membership from the consumer's indirect utility function (that is, verify the Roy equality with respect to commodity *M*).

Problem 2

The standard economic choice model assumes that choice is made from a *set*. Let us construct a model where the choice is assumed to be from a *list*.

Let *X* be a finite "grand set". A *list* is a non-empty finite vector of elements in *X*. In this problem, consider a *choice function* *C* to be a function which assigns to each vector $L = < a_1,\ldots,a_K >$ a single element from $\{a_1,\ldots,a_K\}$. (Thus, for example, the list $< a,b >$ is distinct from $< a,a,b >$ and $< b,a >$). For all $L_1,\ldots,L_m$ define $< L_1,\ldots,L_m >$ to be the list which is the concatenation of the m lists. (Note that if the length of L_i is k_i the length of the concatenation is $\Sigma_{i=1,..,m} k_i$). We say that L' *extends* the list L if there is a list M such that $L' = < L,M >$.

We say that a choice function C satisfies property I if for all $L_1,\ldots,L_m$
$C(< L_1,\ldots,L_m >) = C(< C(L_1),\ldots,C(L_m) >)$.

■A) Interpret property *I*. Give two (distinct) examples of choice functions which satisfy *I* and two examples of choice functions which do not.

■B) Define formally the following two properties of a choice function:

Order Invariance: A change in the order of the elements of the list does not alter the choice and

Duplication Invariance: Deleting an element which appears in the list elsewhere does not change the choice.

Characterize the choice functions which satisfy Order Invariance, Duplication Invariance and condition I.

Assume now that in the back of the decision maker's mind is a value function u defined on the set X (such that $u(x) \neq u(y)$ for all $x \neq y$). For any choice function C define $v_C(L) = u(C(L))$.

We say that C *accommodates a longer list* if whenever L' extends L, $v_C(L') \geq v_C(L)$ and there is a list L' which extends a list L for which $v_C(L') > v_C(L)$.

■C) Give two interesting examples of a choice function which accommodates a longer list.

■D) Give two interesting examples of choice functions which satisfy property I but which do not accommodate a longer list.