Peinceton University
Micro –Economics 501
Mid-term Exam : November, 6th 2000
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Problem 1 (Time preferences)
Let $X=\Re^{+}\times\{0,1,2\dots\}$ where (x, t) is interpreted as getting \$x at time t. Assume that a decision maker has a preference relation on this space with the following properties:

- He is indifferent between getting \$0 at time 0, or at any other time.
- For any positive amount of money he prefers to get it as soon as possible.
- He likes money.
- His preference between (x, t) and (y, t+1) is independent of t (interpret it).
- Continuity.

A. Define the continuity assumption for this model.
B. Show that any preference relation satisfying the above assumptions has a utility representation.
C. Verify that a preference relation which has the form $u(x)\delta^t$ (with $\delta<1$, $u(0)=0$, u continuous and increasing) satisfies all axioms.
D. Formulate a concept "one preference is more impatient than another preference".
E. Discuss a claim that a preference represented by $u_1(x)\,\delta_1^{\,t}$ is more impatient than a preference represented by $u_2(x)\,\delta_2^{\,t}$ if and only if $\delta_1<\delta_2$.

Problem 2 (Indirect utility functions)
Discuss the following consumer. The consumer's initial wealth is w. He likes as much money as possible but for survival he must buy one and only one unit of one and only one of the goods denoted 1,…,K. Commodity k's price is p_k and all prices are less than w (for simplicity, concentrate on a domain where all prices are distinct).

For some reason the consumer prefers not to be seen purchasing the cheapest good and he always purchases the second cheapest good.
A. Define an "indirect utility function" for the consumer.
B. Study the properties of the indirect utility function: monotonicity, continuity and convexity in prices.
C. State the "Roy's equality" for this model and explain why it holds (in every price vector where all prices are distinct).

Problem 3 (Random dictatorship)
Consider the aggregation of preference relations defined on the set {A, B, L} where L is a lottery which assigns A or B with equal probabilities. Assume that all preference relations satisfy the vNM assumptions.
A. Show that there is a social welfare function satisfying the IIA and Pareto axioms which is not dictatorial.
B. Reconcile this fact with Arrow's impossibility theorem.

Problem 4 (Choice with status quo)
This is a "Bonus" question intended only for students who finish the first three questions very early.
Let $X=\Re^K$ be a "grand set". Let c be a function which assigns an element in S to every pair (S,d) where S is a closed and convex subset of X and $d\in X$ (d is not necessarily in S). The function c is interpreted as the choice from S given that d is the "status quo".

A. Formulate the following property of a choice function c: The choice from a set $S\cap T$ given a status quo d can be done invariantly either in one stage, or in two stages, by first selecting an element in S given the original status quo and then selecting a point in T (given the new "status quo").
B. Show that for K=2 there is no such function whereas such a function exists for K=1.