Introduction to Game Theory for Law and Philosophy Students
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Problem Set 5 (Mixed Strategy Nash Equilibrium)

1. A zero sum game

Consider the two-player zero sum game:

	L	R
T	3, –3	0, 0
M	0, 0	3, –3
B	v, –v	v, –v

Calculate the mixed strategy Nash equilibria for this game as a function of v.

2. Air Strike

An attack group commander has a single plane which can strike one of three possible targets. The defender's commander has one anti-aircraft missile battery that can be assigned to one of the targets. The plane will fail in its assignment if it attacks a defended target. The number of enemy troops at the three targets is 5, 4 and 1, respectively. The attacker wishes to maximize the defender's expected losses, while the defender wishes to minimize the expected troop losses.

Analyze the situation as a zero-sum game and calculate its mixed strategy equilibrium.

3. Saving the drown man

n people stand on the beach of a stormy sea watching a person drowning. Each observer is selfish and does not care about the poor victim's fate. Jumping into the stormy sea is dangerous yet each observer wants to be a hero, the only one recognized as brave enough to go to the rescue.

The vNM utilities of each person is:

if he jumps in by himself $+1$ (he is declared a hero)

if he does not jump in 0

if he jumps in but at least one more person jumps in as well -4 (no glory, only risking his life)

What is the symmetric mixed strategy Nash equilibrium?

What is the equilibrium probability that the victim will be saved?

Will the symmetric mixed strategy equilibrium probability that the victim will be saved be higher if more people are present on the beach?

4. A Monitoring Game (more difficult)

The tax authorities aim to maximize the expected sum of money to be collected. It is known that the two large firms have to pay taxes of $\$80m$ and $\$100m$, respectively. The firms are contemplating claiming that they have no income.

The tax authorities have one secret agent who can be planted in advance in one of the firms. The spy will provide the evidence for the tax evasion (if occurs). A fine of 50% (over the tax itself) will be paid by an evading firm if the spy is assigned to this firm. Each firm wants to minimize the expected sum to be paid to the tax authorities.

Thus, we have a three-player game where the tax authorities decide whether to send the spy to firm 1 or 2, and each of the firms has to decide whether or not to evade taxes.

Show that the game has no pure strategy Nash equilibrium.

Calculate its mixed strategy Nash equilibrium.

Would the tax authorities' behavior change if the second firm's profits were $\$1000m$ instead of $\$100m$?