Problem set for Lecture G-09: Repeated Games
Readings: Osborne and Rubinstein Ch 8

Exercise 1: Show that not every strategy in an infinitely repeated game can be executed by a machine with a finite number of states.

Exercise 2: Consider an infinitely repeated game in which the players' preferences are derived from their payoffs in the constituent game using different discount factors. Show that a payoff profile in such a repeated game may not be a feasible payoff profile of the constituent game.

Exercise 3: Consider a two-player infinitely repeated game. For any given machine for player 2 construct a machine for player 1 that yields it a payoff of at least v_1 at any period.

Exercise 4: Let $G =< N, (A_i), (u_i) >$ be a game which has two pure Nash equilibria such that for all i the payoff of the first equlibirum is greater than the payoff of the second equilibrium. Show that any any payoff profile in which every player obtains more than his payoff in the inferior Nash equilibrium of G can be achieved as the average payoff profile in a subgame perfect equilibrium of G^T for T large enough.

Exercise 5: Assume that $G =< N, (A_i), (u_i) >$ has the property that for every i there is a Nash equilibirum of G, a^i, such that $u_i(a^i) > v_i$. Let a^* be a strictly enforceable payoff vector. Show that for T large enough there is a Nash equilibrum of G^T with an average payoff which is within ε from $u(a^*)$.

If too difficult you may make do with the case that $a^i = b$ for all i.

Explain why the Nash equilibrium which you built will tycially not be an SPE?

Exercise 6: Consider an infinte horizon infinite game in which the strategic game G is played between player 1 and an infinite sequence of players, each of whom lives for only one period and is informed of the actions taken in every previous period. Player 1 evaluates sequences of payoffs by the limit of means, and each of the other players is interested only in the payoff that he gets in the single period in which he lives.

a) Find the set of subgame perfect equilibria of the game when G is the Prisoner's Dilemma

b) Show that for the following game, for every rational number x in $[1, 3]$ there is a subgame perfect equilibrium in which player 1's average payoff is x.

	C	D
C	$3, 3$	$0, 0$
D	$4, 0$	$1, 1$

Exercise 7: Consider the infinitely repeated game in which the players' preferences are represented by the discounting criterion, the common discount factor is $\delta < 1/2$, and the constituent game is the game

	A	D
A	$2, 3$	$1, 5$
D	$0, 1$	$0, 1$

Show that $((A, A), (A, A), \dots)$ is not a subgame perfect equilibrium outcome path.