

**Course:** Microeconomics, New York University

**Lecturer:** Ariel Rubinstein

**Exam:** Mid-term, October 2005

**Time:** 3 hours (no extensions)

**Instructions:** Answer the following three questions. The first question seems to me to be more difficult (but I am not sure...). Be concise and accurate as possible.

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**Problem 1** (inspired by Chen, M.K., V.Lakshminarayanan and L.Santos (2005))

In an experiment a monkey is given  $m = 12$  coins. The monkey faces  $m$  consecutive choices. In each instance he gives one coin to either one of two experimenters, one who is holding  $a$  apples and one who is holding  $b$  bananas.

(1) Assume that the experiment is repeated with different values of  $a$  and  $b$  and that every time the monkey trades the first 4 coins for apples and then trades the next 8 coins for bananas. The experimenter claims that the monkey's choices confirm consumer theory.

Show that the above monkey's behavior is indeed consistent with the classical assumptions of consumer behavior (namely, that his behavior can be explained as the maximization of a monotonic, continuous, convex preference relation on the space of bundles).

(2) Assume that later it was observed that when the monkey holds an arbitrary number  $m$  of coins, then independent of  $a$  and  $b$ , he exchanges first 4 coins for apples and then exchanges the remaining  $m - 4$  coins for bananas. Is this behavior consistent with the consumer model?

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**Problem 2**

A consumer lives in a world of  $K$  commodities. He holds classical preferences over those commodities. The goods are split into two categories, 1 and 2, of  $K_1$  and  $K_2$  goods, respectively ( $K_1 + K_2 = K$ ). The consumer receives two types of money:  $w_1$  units of wealth which can be exchanged for goods in the first category only and  $w_2$  units of wealth which can be exchanged only for goods in the second category only.

Define the induced preference relation over the two-dimensional space  $(w_1, w_2)$ . Show that those preferences are monotonic, continuous and convex.

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**Problem 3**

Let  $X$  be a finite set containing at least three elements. In this question a choice function  $C$  assigns for every non-empty subset  $A$  a set  $C(A) \subseteq A$  which is non-empty.

Consider the following "Choice Axiom":

If  $A, B \subseteq X$ ,  $B \subseteq A$  and  $C(A) \cap B \neq \emptyset$ , then  $C(B) = C(A) \cap B$ .

a) Show that the Choice Axiom is equivalent to the existence of a preference relation  $\succeq$  such that  $C(A) = \{x \in A \mid x \succeq a \text{ for all } a \in A\}$ .

b) Consider a weaker axiom:

If  $A, B \subseteq X$ ,  $B \subseteq A$  and  $C(A) \cap B \neq \emptyset$ , then  $C(B) \subseteq C(A) \cap B$ .

Is it sufficient for the above equivalence?

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