Princeton University	
Exam:	**Midterm, November 1998**
The course:	**Econ 501**
The lecturer:	**Ariel Rubinstein**
Length:	**4 hours**
Instruction:	**Answer all questions. Write to the point! You are allowed to use any written material you wish to use.**

Problem 1: Let X be a finite set of alternatives. A decision-maker's behavior is modeled by a function which assigns to any subset $A \subseteq X$ a probabilistic distribution C(A) over A. Denote by C(A)(a) the probability that C(A) assigns to $a \in A$.

Examine the following axiom on the function C:
Axiom I: If a and b are in A, C(A)(a)>0 or C(A)(b)>0 and B is a subset of A which includes both a and b then
C(B)(a)/ [C(B)(a)+C(B)(b)]= C(A)(a)/[C(A)(a)+C(A)(b)].

a) State and prove a proposition of the type: If C satisfies axiom I then there are ????? such that for every subset of X, A, and for every $a \in A$, C(A)(a)=????? (Fill the ???? and prove the proposition)

b) Cosnider a decision-maker who has in mind a fixed probabilistic distribution of orderings over the set X. When he chooses from a subset A, he randomly selects one of the orderings and chooses the element in A, which is the best according to this ordering. Show that the induced C-function does not necessarily satisfy axiom I. That is, there is a probabilistic distribution over the orderings of the set X={a,b,c} such that the induced C-function does not satisfy the axiom.

Problem 2: Consider a consumer in a world with K commodities (with preferences over bundles that satisfy the standard assumptions we make on a consumer). The consumer gets his income in form of a bundle of commodities w and he chooses the best bundle from among the set B(p,w)={x|px=pw}. Given that the consumer's preferences are represented by a utility function u, define V(p,w)=max {u(x)| px=pw}.

a) Interpret the function V.
b) Show that $V(\lambda p,w)$=????
c) Show that V is quasi convex in p.
d) Fix all prices but i, and all quantities of in the initial bundle but w_i. Show that the slope of the indifference curve of V in the two dimensional space where the parameters on the axes are p_i and w_i is $(x_i(p,w)-w_i)/p_i$ where x(p,w) is the solution to the consumer's problem B(p.w).

Problem 3: Who is an economist? Departments of economics are always sharply divided over this question. Investigate the approach that the determination of "who is an economist" should be treated as an aggregation of the views held by the members of the department about this question.

Let $N=\{1,..,n\}$ be a group of individuals ($n\geq 3$). Each $i\in N$ "submits" a set E_i a proper subset of N, interpreted as the set of "real economists" in his view. An aggregation method F is a function which assigns to each profile $(E_i)_{i=1,..,n}$ of proper subsets of N, a proper subset of N, denoted $F(E_1,..,E_n)$, with the interpretation of this set to include all those who are considered to be economists given the profile of views. (Note that we required that all opinions are proper subsets of N.)

Consider the following axioms on F:

Consensus: If $j\in E_i$ for all $i\in N$ then $j\in F(E_1,..,E_n)$ and if $j\notin E_i$ for all $i\in N$ then $j\notin F(E_1,..,E_n)$.

Independence: If $(E_1,..,E_n)$ and $(E'_1,..,E'_n)$ are two profiles of views so that for all $i\in N$, [$j\in E_i$ if and only if $j\in E'_i$] then [$j\in F(E_1,..,E_n)$ if and only if $j\in F(E_1,..,E_n)$].

a) Interpret the two axioms.

b) Show one aggregation method which satisfies C and not I and one which satisfies I and not C.

c) Show that the only aggregation methods which satisfy the above two axioms are those for which there is a member j such that $F(E_1,..,E_n)\equiv E_j$. (This is the hardest part of the exam. Construct a proof similar to that of Arrows' impossibility theorem).