

Please answer as many questions as you can. Be focused, short and accurate.

1. A decision maker forms preferences over the set  $X$  of all possible distributions of a population over two locations. An element of  $X$  is a pair of  $(x_1, x_2)$  where  $x_i \geq 0$  and  $x_1 + x_2 = 1$ . The decision maker has two principles in mind:
- He wants the preference to have the property that if  $x \succeq y$  then mixing each of the populations with a third population distributed  $z$ , in proportions  $\alpha$  and  $(1-\alpha)$  creates a new pair of distributions  $x'$  and  $y'$  so that  $x' \succeq y'$  as well.
  - The decision maker is indifferent between a distribution which is fully concentrated in location 1 and one which is fully concentrated in location 2.
    - Show to the decision maker that the only preference which is consistent with the two axioms is the degenerate preference which ranks all distributions equally!!
    - The decision maker claims you must be wrong as his preference is represented by a utility function  $|x_1 - 1/2|$ . Show him that he is wrong.

2. An event which could have happened with probability  $1/2$  either occurred or not. A firm has to supply a report of the form “the event occurred” or “the event did not occur”. The quality of the report, (the product of the firm),  $q$ , is the probability that the report is correct. For preparing the report the firm employs  $k$  experts (input), each of them gets an independent signal whether the event occurred or not which is correct with probability  $1/2 < p < 1$ .
- Calculate the production function  $q=f(k)$  for (at least)  $k=1,2,3$ .
  - We say that a “discrete” production function is concave if the marginal products are non-increasing. Is the production function concave?
  - Explain why it is true that if  $f$  were concave then the firm who maximizes profits with respect to given prices of “quality” and “workers” would choose a level  $k^*$  so that the  $k^*$ ’th worker is the last worker for which the “marginal revenue of production” is above the price of a single worker.
  - Is this important conclusion true in our case?

3. A consumer with wealth  $w$ , must buy one of three books. The prices of the books are  $p_1, p_2$  and  $p_3$ . Assume that all prices are below  $w$ . The behavior of the consumer is as follows: He first compares the first two prices. If  $p_1 \leq p_2$  then he purchases book 1 (without looking at  $p_3$ ). If  $p_1 > p_2$  he compares  $p_2$  with  $p_3$  and purchases book 2 if  $p_2 \leq p_3$  and book 3 if  $p_2 > p_3$ . He spends all his remaining wealth on “all other goods”.
- What is the demand function of the consumer?
  - Does the consumer satisfy the assumptions about the rational man?
  - Would you view the function  $V(p_1, p_2, p_3, w) = w - p_{i^*}$  (where  $i^*$  is the good that consumer purchases at the price vector  $(p_1, p_2, p_3)$  and wealth  $w$ ) as an indirect utility function?
  - Does  $V$  satisfy the Roy’s equality?