Lecture Notes in
Microeconomic Theory

Corrections and Updates

February 5^{th}, 2006

Lecture One

Page 5, line 7 from top

At the end of the line, replace "if $x \geq y - 1$" with "if $x \geq y + 1$".

Page 6, line 4 from bottom

Insert the emphasized text: "...than the existence of a one-to-one correspondence *(i.e., a one-to-one and onto function)*: the correspondence..."

Page 7, line 4 from top (after table)

Replace the word "correspondence" with the word "function".

Page 7, last paragraph, 2nd and 3rd lines

The paragraph starts with "The following observations complete the proof...". In line 2 replace "from" with "between". In the next line replace "onto" with "and".

Page 8, 5$^{\text{th}}$ paragraph, 1st line

The paragraph starts with "To see that $Translation$ is indeed a one-to-one correspondence,...". Replace the word "correspondence" with the word "function".

Page 8, one paragraph before the last

The paragraph starts with "We still have to check that f statisfies the transitivity condition." Replace every occurence of "F" with "f". For example, the start of the 2nd line should read "If $f(x, y) = x \succ y$ and $f(y, z) = y \succ z$,...". There are 5 replacements to be made.

Lecture Three

Page 38, Problem 4

In line 2 of the problem insert ", separately," (between commas) after the word "and" and before the word "he", so that the sentence reads "...over the set X and, separately, he assigns to each x...".

In lines 5,6 and 7, exchange the term "most popular" for "most common".

Lecture Four

Page 48, after the proof, before "Differentiable Preferences"

The following new text should enter at this point.

Claim:

Any continuous preference relation $\succsim$ on $\Re_+^K$ satisfying strong monotonicity and quasi-linearity in all commodities can be represented by a utility function of the form $\sum_{k=1}^K \alpha_k x_k$.

Proof:

Here I present a proof for the case of $K = 2$ only. The general proof for any K is left for the problem set.

Using the previous claim, we have that the preference relation over the bundle space is represented by the function $u(x_1, x_2) = x_1 + v(x_2)$ where $(0, x_2) \sim (v(x_2), 0)$. Let $(0, 1) \sim (\alpha_2, 0)$. It is sufficient to show that $v(x_2) = \alpha_2 x_2$.

Assume that for some x_2 we have $v(x_2) > \alpha_2 x_2$ (a similar argument applies for the case $v(x_2) < \alpha_2 x_2$). Choose two integers S and T such that $v(x_2)/\alpha_2 > S/T > x_2$. Note that $(0, x_2) \sim (x_1, 0)$ implies that, for any n, all points $(n_1 x_1, n_2 x_2)$ for which $n_1 + n_2 = n$ reside on the same indifference curve. We prove this by induction on n, using the fact that $((n-1)x_1, 0) \sim ((n-2)x_1, x_2)$ implies $(nx_1, 0) \sim ((n-1)x_1, x_2)$ and $(n_1 x_1, (n_2 - 1)x_2) \sim ((n_1 - 1)x_1, n_2 x_2)$ implies $(n_1 x_1, n_2 x_2) \sim ((n_1 - 1)x_1, (n_2 + 1)x_2)$.

Thus, $(0, Tx_2) \sim (Tv(x_2), 0)$ and $(0, S) \sim (S\alpha_2, 0)$. However, since $S > Tx_2$ we have $(0, Tx_2) \prec (0, S)$, and since $Tv(x_2) > S\alpha_2$ we have $(Tv(x_2), 0) \succ (S\alpha_2, 0)$ which is a contradiction.

Page 50, Problem 5

The current problem should be replaced in its entirety by the following:

Complete the proof of the claim that any continuous preference relation on $\mathbb{R}_+^K$ satisfying strong monotonicity and quasi-linearity in all commodities can be represented by a utility function of the form $\sum_{k=1}^K \alpha_k x_k$.

Lecture Five

Page 56, 1$^{\text{st}}$ paragraph

The 1st paragraph ("If x^* is not a solution,...") should be replaced in its entirety by the following paragraph:

> If x^* is not a solution, then there is a bundle y such that $py \leq px^*$ and $y \succ x^*$. By continuity we can assume that $y_k > 0$ for all k. Also by continuity, there is a bundle z with $z_k < y_k$ for all k such that $z \succ x^*$. Let $1 \geq \lambda > 0$. By convexity, $\lambda z + (1-\lambda)x^* \succsim x^*$ and by montonicity $\lambda y + (1-\lambda)x^* \succ \lambda z + (1-\lambda)x^*$. Thus, any small move in the direction $(y - x^*)$ is an improvement, and by differentiability, $v(x^*)(y-x^*) > 0$.

Furthermore, in the equalities of the 2nd paragraph, the leftmost inequality should be weak: "$0 \geq p(y - x^*) = \sum$..."

Page 58, 1st paragraph, 3rd line

The part enclosed in parantheses "(If the function... ...is continuous.)" should be replaced by the following:

> (Let $f(x)$ be a continuous function over X. Let A be a subset of some Euclidean space and B a function that attaches to every $a \in A$ a compact subset of X such that its graph, $\{(a,x) |\ x \in B(a)\}$, is closed. Then the graph of the correspondence h from A into X, defined by $h(a) = \{x \in B(a) \mid f(x) \geq f(y) \text{ for all } y \in X\}$, is closed.)

Page 58, after the proof, before "Rationalizable Demand Functions"

The following new text should enter at this point (not as part of the proof).

> *Comment*: The above proposition applies to the case in which for every budget set there is a unique bundle maximizing the consumer's preferences. A similar claim can be proven for the case in which some budget set has more than one maximizer: If $\succsim$ is a continuous preference, then the set $\{(x,p,w) \mid x \succsim y \text{ for every } y \in B(p,w)\}$ is closed.

Page 62, Strong Axiom of Revealed Preference (SA)

The provided definition of the SA (lines 8-10 from the top) should be replaced by the following paragraph:

> If $(x^n)_{n=1,..,N}$ is a sequence of distinct bundles and $(B(p^n, w^n))_{n=1,..,N}$ is a sequence of budget sets so that
> - for all n, the bundle x^n is chosen from $B(p^n, w^n)$ and
> - $x^{n+1} \in B(p^n, w^n)$ for all $n \leq N - 1$,
>
> then $x^1 \notin B(p^N, w^N)$.

Furthermore, the paragraph following the definition ("The Strong Axiom is basically equivalent to...") should be replaced by the following:

> The Strong Axiom states that the relation $\succ$, derived from revealed behavior, is acyclical. This leaves open the question of whether $\succ$ can be extended into preferences. (Note that its transitive closure still may not be a complete relation.) The fact that it is possible to extend the relation $\succ$ into a full-fledged preference relation is a well known result in Set Theory. In any case, the SA is somewhat cumbersome and using it to determine whether a certain demand function is rationalizable may not be a trivial task.

Page 62, Decreasing Demand, 1st paragraph

The paragraph starts with "The consumer model discussed so far...". The whole paragraph should be replaced by the following text:

> A theoretical model is usually evaluated by the reasonableness of its implications. If we find that a model yields an absurd conclusion, we reconsider its assumptions. However, note that alarm bells should also go off when we find that a model fails to yield highly intuitive properties which may indicate that we have assumed "too little".
>
> For example, in the context of the consumer model, we might wonder whether the intuition that demand for a good falls when its price increases is valid. We shall now see that the standard assumptions of rational consumer behavior do not guarantee that demand is decreasing. The following is an example of a preference relation which induces demand that is nondecreasing in the price of one of the commodities.

Lecture Six

Page 72, 3^{rd} paragraph after numbered list

The paragraph starts with “If it takes t^* for *to* the turtle...” . Delete the emphasized “to”.

Lecture Eight

Page 88, 7^{th} line from top

Insert a comma between “if it is not” and “is *not* equivalent”, so that the sentence reads “A “lottery” in which you have z_1 if it is raining and z_2 if it is not**,** is not equivalent to the ‘lottery” in which you have z_1 if it is not raining and z_2 if it is.”

Page 89, after bulletpoints

Before the paragraph starting with “The richness of examples calls for...”, after the bullets (not as part of the bullets), the following text should enter:

> Note that the above examples constitute ingredients which could be combined in various ways to form an even richer class of examples. For example, one preference can be employed as long as it is “decisive” and a second preference can be used to break ties when it is not.

Page 98, lines 9 and 13

At the end of each of these lines appear 4 dots when only 3 are in order. Delete a dot from each.

Lecture Nine

Page 108, figure 9.4

The drawing contains an equation representing the diagonal line connecting the two axes, in which the letter "c" should be replaced with a "t", so that it reads "$px_1 + (1-p)x_2 = t$".

Page 109, line 9 from bottom to page 110, 2nd line from top

The current text from "We conclude that the function u,..." on page 109 to "...we have $\psi_t(x_1) = \psi_0(x_1 - t) + t$." on page 110, should be replaced by the following text:

> Using the formula for the sum of a geometric sequence, we conclude that the function u, defined on the $\Delta - grid$, must equal $a - b(\frac{1-q}{q})^{\frac{x}{\Delta}}$ for some a and b.
>
> Let us now return to the case of $Z = \Re$ and look at the preferences over the restricted space of all lotteries of the type $(x_1, x_2) = px_1 \oplus (1-p)x_2$ for some arbitrary fixed probability $p \in (0,1)$. Denote the indifference curve through (t,t) by $x_2 = \psi_t(x_1)$. Thus, $[t] \sim px_1 \oplus (1-p)\psi_t(x_1)$. Since $\succsim$ exhibits constant absolute risk aversion, it must be that $[0] \sim p(x_1 - t) \oplus (1-p)(\psi_t(x_1) - t)$ and thus $\psi_0(x_1 - t) = \psi_t(x_1) - t$, or $\psi_t(x_1) = \psi_0(x_1 - t) + t$. In other words, the indifference curve through (t,t) is the indifference curve through $(0,0)$ shifted in the direction of (t,t).

Lecture Ten

Page 117, Condition I^*

To give the definition the same look and feel as the previous two definitions, rewrite it as (the meaning remains the same):

> For all $a, b, c, d \in X$, and for any two profiles $(\succ_i)_{i \in N}$ and $(\succ'_i)_{i \in N}$,
> if for all i, $a \succ_i b$ iff $c \succ'_i d$, then $a \succsim b$ iff $c \succsim' d$.

Page 119, end of page

After the proof of Arrow's Theorem, insert the following comment:

> *Comment:* Proving the theorem with conditions Par and IIA only requires a few more steps. First, for every two alternatives x and y, define the notion "G is decisive with regard to (x, y)" and "G is almost decisive with regard to (x, y)". Then, proceed through the following steps:
>
> • If G is almost decisive with regard to (x, y), then G is almost decisive with regard to (x, z). (Consider the profile in which for every $i \in G$, $x \succ_i y \succ_i z$ and for every $i \notin G$, $y \succ_i z \succ_i x$).
>
> • If G is almost decisive with regard to (x, y) then G is almost decisive with regard to (y, z). (Consider the profile in which for every $i \in G$, $y \succ_i x \succ_i z$ and for every $i \notin G$, $z \succ_i y \succ_i x$).
>
> • If G is almost decisive with regard to (x, y), then G is decisive with regard to (x, y).
>
> • If G is decisive with regard to (x, y) and $|G| \geq 2$, then there exists $G' \subset G$ which is decisive with regard to (x, y).
>
> • For every x and y, there is an individual $i(x, y)$ such that $\{i(x, y)\}$ is decisive with regard to (x, y).
>
> (The proof of the last three steps is very similar to that given above.)
>
> • Verify that $i(x, y) = i(x', y')$ for every (x, y) and (x', y').

Page 123, Problem Set 10, Problem 4

This is the correct form of the axioms defined on F:

> • *Consensus*: If $j \in E_i$ for all $i \in N$, then $j \in F(E_1, \dots E_n)$ and if $j \notin E_i$ for all $i \in N$, then $j \notin F(E_1, \dots E_n)$.
>
> • *Independence*: If $(E_1, \dots, E_L)$ and $(G_1, \dots, G_n)$ are two profiles of views so that for all $i \in N$, $[j \in E_i$ iff $j \in G_i]$, then $[\ j \in F(E_1, \dots, E_n)$ iff $j \in F(G_1, \dots, G_n)]$.

Review Problems

Page 129, at the end

Add the following two review problems:

Problem 12 (NYU 2005, inspired by Chen, M.K., V. Lakshminarayanan and L. Santos (2005))

In an experiment, a monkey is given $m = 12$ coins which he can exchange for apples or bananas. The monkey faces m consecutive choices in which he gives a coin either to an experimenter holding a apples or another experimenter holding b bananas.

1. Assume that the experiment is repeated with different values of a and b and that each time the monkey trades the first 4 coins for apples and the next 8 coins for bananas.

Show that the monkey's behavior is consistent with the classical assumptions of consumer behavior (namely, that his behavior can be explained as the maximization of a montonic, continuous and convex preference relation on the space of bundles).

2. Assume that it was later observed that when the monkey holds an arbitrary number m of coins, then, irrespective of the values of a and b, he exchanges the first 4 coins for apples and the remaining $m - 4$ coins for bananas. Is this behavior consistent with the rational consumer model?

Problem 13 (NYU 2005)

A consumer has classical preferences in a world of K goods. The goods are split into two categories, 1 and 2, of K_1 and K_2 goods respectively ($K_1 + K_2 = K$). The consumer receives two types of money: w_1 units of money which can only be exchanged for goods in the first category and w_2 units of money which can only be exchanged for goods in the second category.

Define the induced preference relation over the two-dimensional space (w_1, w_2). Show that these preferences are monotonic, continuous and convex.

References

Page 131, 1st paragraph

This is the correct URL address for the online version of the book: http://arielrubinstein.tau.ac.il/micro1/ .

Page 131, between references

Add the following reference in appropriate alphabetical order:

> Chen, M.K., V. Lakshminarayanan and L. Santos (2005). "The Evolution of our Preferences: Evidence from Capuchin-Monkey Trading Behavior", mimeo.